

# Floating ice plate failure due to its thermal expansion at the surface

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## ABSTRACT

The problem of a floating ice sheet failure caused by stresses induced in ice by temperature changes at its top surface is investigated. The ice cover is modelled as a plate of uniform thickness, which is laterally constrained at its edges by rigid walls, and is assumed to deform, and ultimately fail, by the mechanism of creep buckling. The floating plate is subjected to in-plane compressive stresses developing in ice to prevent its lateral expansion due to heating, and is transversely (vertically) bent by the forces acting at its base and caused by the reaction of underlying water. The sea ice is treated as a material whose elastic and viscous properties depend on its porosity and current temperature, and therefore vary with the depth of ice. The results of simulations, carried out for a variety of ice plate spans, thicknesses and temperature-change scenarios, illustrate the evolution of creep buckles in the plate prior to its failure, and show the time variation of the magnitudes of forces exerted by ice on the constraining walls.

## 1. Introduction

Floating sea, lake or river ice, like any material, expands when subjected to heating. Surprisingly, little attention has been paid yet to this phenomenon and its consequences for civil engineering, despite the fact that thermally-induced forces developing in ice due to its in-plane expansion, especially in cases when ice is constrained in lateral directions, can reach magnitudes which are dangerous for the safety of engineering structures. This work addresses one of the problems that can be of interest to civil engineers. Namely, the forces which a floating ice sheet exerts on a structure due to ice heating at its top surface are determined, and their time variation during the heating process is investigated. The analysis is carried out on the assumption that the forces developing in ice and then transferred to a structure originate from the elastic reaction of the medium to its thermally-induced expansion being prevented by constraints imposed on the ice deformation by the structure walls. Once in-plane elastic compressive forces have occurred in the ice cover, the ice starts to deform by viscous creep, giving rise to its off-plane buckling which ultimately leads to the ice failure when the ice flexural strength is exceeded.

The topic of a floating ice sheet behaviour under the action of in-plane axial forces and off-plane transverse loads due to the underlying water reaction has been analysed by a number of investigators. The problem of elastic buckling of a floating plate was solved in the papers by Kerr (1978); Nevel (1980) and Sanderson (1988), in which approximate estimates for the magnitudes of the buckling forces, derived analytically,

are given. The approximate formulae proposed in these papers were subsequently refined and corrected by Staroszczyk (2002), on the basis of results obtained from finite-element calculations. Some relevant analytical results can be also found in the work by Kerr and Palmer (1972), and experimental data on the elastic buckling of ice have been reported by Sodhi et al. (1983). A more realistic problem of creep buckling of floating ice was treated by Sjölin (1985); Sanderson (1988); Staroszczyk (2003) and Staroszczyk and Hedzielski (2004). In none of the above papers the thermally-induced loads and their effects on the ice plate behaviour were investigated.

In the present paper, an attempt is made to examine how the mechanical response of ice is affected by the changes in the temperature field on the ice surface. Hence, thermally-induced axial stresses are accounted for in the balance of all the forces acting on a floating plate, and the effects of these thermal stresses on the mechanism of the ice creep buckling and the subsequent ice failure are studied. Of prime interest are the magnitudes of the forces exerted by ice on the walls constraining the plate in the horizontal direction, and the evolution of these forces as the ice creep deformation proceeds until the instant of the ice flexural fracture. The evolution of the thermally-induced forces within the ice plate is not only due to the rise in temperature at the ice top surface, but also due to the progressive vertical heat transfer through the ice from its top to the bottom. The latter process results in the variation of the elastic and viscous properties of the material, which are assumed to be temperature-dependent; therefore, the plate cannot be treated as a homogeneous, since its mechanical properties vary with depth. Further,

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the evolution of the plate deflection surface with increasing thermally-induced axial loads is investigated, with the analysis of the characteristics of buckles which grow in time as the ice deformation progresses.

The analysis of the floating ice sheet behaviour is carried out by applying the classical theory of thin plates resting on an elastic foundation (Timoshenko and Woinowsky-Krieger, 1959). The equations formulating the problem considered are given in Section 2, together with the relations describing the temperature-dependence of the elastic and viscous properties of ice. In Section 3, the method of solution of the equation governing the creep deformation of the plate is presented, and the analysis of fundamental properties of this solution is carried out. Section 4 is devoted to the presentation of the results of numerical simulations carried out for assumed temperature variation scenarios. The results illustrate the effects of the plate thickness and length on the creep behaviour of the ice and the magnitudes of the forces acting on the walls constraining the ice. Finally, some conclusions are drawn in Section 5.

## 2. Problem formulation

The problem under consideration is depicted in Fig. 1. An ice sheet is modelled as a plate of uniform thickness  $h$  and length  $L$ , and is assumed to be constrained by vertical rigid walls at the plate ends at  $x = 0$  and  $x = L$ . The vertical coordinate axis  $z$  is directed downwards, with  $z = 0$  at the upper surface of the plate, and  $z = h$  at the base of the plate.

The plate deflection surface is denoted by the function  $w(x, t)$ , with  $t$  denoting time, and  $w > 0$  for the downward deflection. The plate of ice is floating on the surface of underlying water, which exerts an elastic reaction on the ice, proportional to the plate deflection. The ice top surface is subjected to the action of varying in time temperature  $T(t)$ , with the ice at the base ( $z = h$ ) being at the melting point temperature  $T_m$  all the time, and  $T < T_m$  throughout the ice plate. It is assumed that at the initial time  $t = 0$  the plate is stress-free; that is, it is in equilibrium under an initial distribution of temperature in the plate. For simplicity, a plane-strain problem is analysed here, so that the ice plate can be treated as a beam of uniform width, with its elastic flexural rigidity adjusted accordingly to account for the zero deformations in the lateral direction normal to the plane  $Oxz$ .

In accordance with the standard theory of thin plates (Timoshenko and Woinowsky-Krieger, 1959), it is assumed that the plate deflections are small (that is, they are of the order of the plate thickness), and the plate cross-sections which are normal to the middle plane in the undeformed state remain plane and normal to the middle surface of the deformed plate. The plate is bent in the transverse (vertical) directions by loads coming from the reaction of the underlying water base (these loads are denoted by  $q$  in Fig. 1 b), and their result is either the lifting of the plate or its depression from the floating equilibrium state. Besides the bending, the plate is also subjected to the action of in-plane axial forces, which in our case are generated by the changes in the ice temperature.

Let denote the internal forces acting per unit width of the plate as  $M$ ,  $Q$  and  $N$  (see Fig. 1 b), where  $M$  is the bending moment,  $Q$  is the vertical shear force, and  $N$  is the normal (tensile) force. Then, neglecting the own weight of the plate and the inertia forces due to negligibly small velocities of ice, the equilibrium balances of forces in the  $z$ -axis direction and

the bending moments acting on an infinitesimal plate element give the relations

$$\frac{\partial Q}{\partial x} + N \frac{\partial^2 w}{\partial x^2} = -q, \quad \frac{\partial M}{\partial x} = Q. \tag{1}$$

The transverse distributed load  $q$  is equal to the underlying water reaction. The latter is assumed to be elastic and linearly proportional to the local plate deflection  $w$  (the Winkler–Zimmerman-type foundation); hence, the load  $q$  is expressed by

$$q = -\rho_w g w, \tag{2}$$

where  $\rho_w$  is the water density and  $g$  is the acceleration due to gravity. By eliminating now the shear force  $Q$  from equation (1) and using (2), one obtains the differential equation

$$\frac{\partial^2 M}{\partial x^2} + N \frac{\partial^2 w}{\partial x^2} = \rho_w g w, \tag{3}$$

where  $M$  and  $N$  are the functions of  $x$  and  $t$ . The above internal forces are defined in terms of the normal stresses  $\sigma_{xx}$  by the relations

$$N = \int_0^h \sigma_{xx} dz, \quad M = \int_0^h \sigma_{xx} z dz. \tag{4}$$

The stresses  $\sigma_{xx}$  in equation (4) are related to the plate deformations and their rates by constitutive relations describing the physical properties of the material and its response to loading. It is well known (Mellor, 1980; Sanderson, 1988) that at typical stress levels occurring in sea ice, equal to about 1 MPa, the creep strains in ice exceed the elastic ones in a matter of about 1 min from the start of loading. Thus, the dominant mode of deformation in floating ice is viscous creep, and hence this type of the material response is considered in the analysis of the problem under investigation. However, the elastic behaviour of ice is also important, since the axial force causing the buckling of the plate arises due to the elastic reaction of the body to heating. In order to describe the viscous response of ice, a constitutive law of the Reiner–Rivlin type (Schulkes et al., 1998; Morland and Staroszczyk, 1998) is employed:

$$\sigma_{ij} = (\zeta - \mu) D_{kk} \delta_{ij} + 2\mu D_{ij} \quad (i, j, k = 1, 2, 3), \tag{5}$$

where the summation convention for repeated indices applies. In (5),  $\delta_{ij}$  is the Kronecker symbol,  $\zeta$  and  $\mu$  denote the bulk and shear viscosities of ice, respectively, and  $D_{ij}$  are the components of the strain-rate tensor. The latter components are defined by

$$D_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \quad (i, j = 1, 2, 3), \tag{6}$$

where  $v_i$  denote the ice velocity vector components. Here, of interest is only the strain-rate component  $D_{xx}$  due to the plate bending, and this is given in terms of the curvature-rate of the plate deflection surface by the relation

$$D_{xx} = \dot{\kappa} (z - z_0) = -\frac{\partial^2 \dot{w}}{\partial x^2} (z - z_0). \tag{7}$$

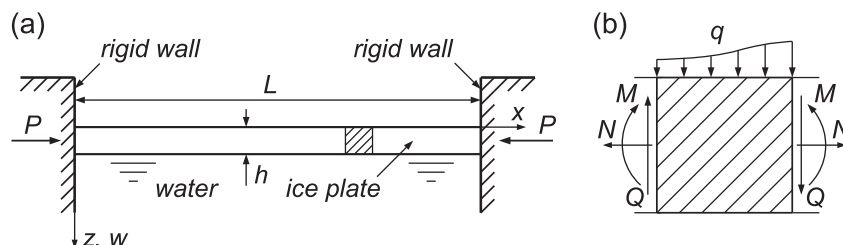


Fig. 1. Floating ice plate of thickness  $h$  and span  $L$  constrained by vertical rigid walls: (a) plate vertical cross-section, (b) definition of internal forces.

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