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Hydrodynamics and near trapping effects in arrays of multiple elliptical cylinders in waves

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ABSTRACT

The purpose of the present study is the investigation of the hydrodynamic interactions induced by arrays of elliptical cylinders subjected to regular waves. The solution methodology employs linear potential theory and is based on pure analytic considerations. The interaction phenomena are approached using the Mathieu functions addition theorem which converts elliptical harmonics from one elliptic coordinate system to a remote elliptic coordinate system. The followed approach resembles the 'direct' solution methodology which is used for the analytical solution of the diffraction problem by arrays of circular cylinders. The 'direct' approach allows, among others, the construction of a linear matrix equation for the calculation of the expansion coefficients of the diffraction component(s) which accordingly is used to trace the wavenumber(s) under which trapped modes may be stimulated to induce wave trapping in the array, and reduction of the energy radiated to the far-field. The numerous computations which are performed, specifically verify that any array of elliptical cylinders, in accord with arrays of circular cylinders, is potentially a wave trapping configuration, which is connected mainly i) with sharp amplifications in all modes of hydrodynamic loading (both forces and moments) and ii) strong free-surface elevations in the liquid regions between the cylinders and on the cylinders' surfaces accordingly.

1. Introduction

When arrays of multiple bodies are subject to incident waves, very interesting hydrodynamic phenomena arise. Multi-body arrangements subjected to the action of propagating waves, induce, by default, continuous reflections of the incoming waves yielding intriguing phenomena such as rather complicated variations of the hydrodynamic loading and hydrodynamic resonances which are reflected as peaks in loading and strong free-surface elevations between the cylinders and also on the wetted surfaces of the bodies. With regard to the former, the complications in hydrodynamic loading are caused mainly because additional parameters are incorporated into the problem, namely the number, arrangement, orientation and size(s) of the bodies (in connection with the water depth). As far as the hydrodynamic resonances are concerned, these are also due to the hydrodynamic interactions and they are encountered when the geometry of the arrangement allows open liquid spaces between the bodies. Those effects are commonly referred as trapped and near trapped modes and they are detected at specific wavenumbers which eventually depend on all parameters involved in the problem.

Such problems have been considered by researchers for several decades. Indeed, multi-body arrangements are detected in a series of engineering designs. For example, sea bridges' piers are an array of usually circular bodies. Another application is floating islands that are designed as plates supported above the water surface by many floating bodies and are used for example as airports. The most common example is however, the columns of tension-leg or bottom-seated offshore structures. These multi-column platforms can exhibit extreme wave loading also due to the occurrence of trapped modes. Furthermore, the increased surface elevation, that is also very often associated with trapped modes, can lead to the damage of the lower deck of such structures due to a false air-gap design. These effects have been also observed experimentally (See for example Swan et al., 1997). On the other hand, wave trapping is by all means not always something we need to avoid. For example, the energy that multi-body configurations can trap in certain cases, while interacting with surface waves, can be exploited by wave-power devices in order to increase their productiveness and efficiency.

Any geometry or arrangement that creates a liquid space between walls (i.e. surfaces where a Neumann condition applies) is a potentially trapped mode structure. A notable single-body geometry that induces

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trapped modes is the elliptical torus (McIver and Porter, 2002), which is referred often as the McIver toroid (Newman, 1999). A circular toroidal structure approximated by ring elements was investigated by Mavrakos (1997) who reported the existence of peaks in the transfer functions of both the exciting forces and the hydrodynamic coefficients. Other examples of wave trapping structural arrangements are the 'moonpools', which were examined for instance by Molin (2001) and Mavrakos and Chatjigeorgiou (2009).

Clearly, wave trapping may also be induced by multi-body arrangements. In that respect the only geometry that has been considered, at least analytically, is that of the circular cylinder. Arrays of cylinders have been studied by several authors, without focusing always on wave trapping phenomena. In that respect, the proper methodology, known as the 'direct' method, was first derived by Záviška (1913) and was rediscovered by Spring and Monkmeyer (1974). Mingde and Yu (1987) literally re-employed Spring and Monkmeyer's (1974) approach for a pair of bottom-seated cylinders. McIver and Evans (1984) based their work on the study of Simon (1982). They used an approximated method assuming large spacing between the cylinders and reported favourable agreement with the results presented by Spring and Monkmeyer (1974). Linton and Evans (1990) elaborated the method further by introducing the linear system that provides the unknown expansion coefficients of the diffraction potential(s) into the formulae of the potential(s) yielding an elegant compact form.

The hydrodynamics of arrays of cylinders have been also studied through an alternative approach, known as the method of 'multiple scattering', the foundations of which were provided by Twersky (1952). In hydrodynamics, it was first employed by Ohkusu (1974). Examples of studies by this method are those due to Kagemoto and Yue (1986) who employed the physical idea of Ohkusu (1974) but algebraically used Simon's (1982) matrix formulation, and the works of Mavrakos and Koumoutsakos (1987), Kagemoto and Yue (1993), Mavrakos and Kalofonos (1997), Yilmaz and Incecik (1998), Yilmaz (2004) and Child and Venugopal (2010).

Evans and Porter (1997a, 1999) as well as Meylan and Eatock Taylor (2009) investigated possible trapping and near trapping effects by means of the method of Linton and Evans (1990). Maniar and Newman (1997) based a great part of their study on the wave trapping by a long array of cylinders (nine in particular) on the concerned 'direct' method as well. Evans and Porter (1997b) extended their efforts to trace possible trapped modes in multiple cylinders in a channel. Wave trapping effects in arrays of cylinders under random wave spectra were investigated by Walker and Eatock Taylor (2005) and Grice et al. (2013), while Malenica et al. (1999) extended the investigation on wave trapping for an array of equally spaced, identical circular cylinders to second-order in wave steepness. The wave trapping by an array of truncated cylinders was considered by Siddorn and Eatock Taylor (2008) and Wolgamot et al. (2015). Independently of the theoretical studies, the fact that wave trapping may occur in multi-body arrangements, was observed in laboratory experiments by Ohl et al. (2001).

The above discussion evinces that wave trapping effects by arrays of circular cylinders using analytical models, are well treated in the literature. Admittedly, the same does not hold for different geometries, such for example multiple elliptical cylinders, although it should be mentioned that for more complex geometries, such as multi-column gravity platforms, semisubmersibles or tension leg platforms and other trapping structures, numerical diffraction codes have been employed by several researchers. The development of an analytical solution methodology for the water wave diffraction problem by arrays of elliptical cylinders, and accordingly the investigation of potential wave trapping phenomena, is feasible due to the existence of separable solutions of the Laplace equation in elliptic coordinates. However, relevant studies are scarce in the literature and mostly concern isolated bodies. Chen and Mei (1971) solved the scattering and radiation problems of water waves by a bottom-seated, surface-piercing elliptical cylinder using the separable solutions of the Laplace equation. Later on, Chen and Mei (1973) investigated the hydrodynamic loading on a stationary platform of elliptical shape partially immersed in the free surface. Williams (1985a) presented two approximate solutions to the scattering problem. One based on the extension of the exact solution of Chen and Mei (1971) for small values of the elliptic eccentricity, the other based on the integral equation technique involving the application of Green's theorem. The same author (Williams, 1985b) extended his method to include the case of a submerged elliptical structure resting on the sea bed. Williams and Darwiche (1988, 1990) used exact solutions to investigate respectively the scattering and radiation problems by truncated elliptical cylinders, while Zhang and Williams (1996a, 1996b) tackled the same problems by a fully submerged elliptical disk.

The hydrodynamic interactions by arrays of elliptical cylinders were first considered by Chatjigeorgiou and Mavrakos (2010) who achieved an analytical solution by employing the so-called addition theorem for Mathieu functions. Numerical results for the exciting forces were presented only for a pair of cylinders. Accordingly, Chatjigeorgiou (2011) presented an analytical solution for groups of elliptical and circular cylinders while in the later study of Chatjigeorgiou (2013) the methodology was enhanced to account for cylinders' truncation. Chatjigeorgiou and Mavrakos (2010) results were accordingly used by Chen and Lee (2012) to validate their own calculations obtained via a boundary integral equation approach. The same numerical approach was used to trace near trapping occurrences induced by arrays of four cylinders (Chen and Lee, 2013).

The primary task of the present study is the investigation of the wave trapping phenomena due to the hydrodynamic interactions by arrays of elliptical cylinders. The boundary value problem defined by multi-body arrangements is further elaborated to provide compact, elegant formulae for the potential and all hydrodynamic loading components, i.e. surge, sway forces, roll, pitch and yaw moments. Three different arrays are considered, composed by four, five and nine cylinders respectively. In the latter case, two different orientations of the semimajor axes relatively to the wave propagation were considered; one perpendicular and one collinear. Attention is paid to situations of in-phase and out-of-phase loading which may occur in cases of specific orientations in terms of the direction of the incoming waves.

Special consideration is given to wave trapping phenomena and their correlation with peaks in hydrodynamic loading and strong free-surface elevations in the open liquid space between the elliptical cylinders in critical wavenumbers. The occurrence of the trapped modes is verified through the homogeneous solution of the linear matrix equation that is constructed to provide the even and odd expansion coefficients of the components of the diffraction potential(s).

2. The hydrodynamic problem

A group of elliptical cylinders, which are fixed on the bottom and exceed the free surface, is considered. The array is subjected to the action of regular waves of amplitude A and circular frequency ω. The bottom is considered flat and horizontal while the water depth is equal to h. The fluid is inviscid and incompressible and the flow irrotational allowing the use of the linear potential theory. The flow field in the three-dimensional Euclidean space is governed by the linear velocity potential $\Phi(x, y, z, t)$ which is written as

$$\Phi(x, y, z, t) = Re\{\phi(x, y, z)e^{-i\omega t}\}, \tag{1}$$

where Re denotes the real part of component in the brackets and t is time. As usual, the spatial complex potential $\phi(x, y, z)$ should satisfy the Laplace equation

$$\nabla^2\phi = 0, \tag{2}$$

everywhere in the fluid domain, the linearized free-surface boundary condition