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On the buoyancy load formulation for geometrically nonlinear analysis of flexible marine risers

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ABSTRACT

A proper buoyancy load formulation that complements the continuum formulation with incorporated beam theory in geometrically nonlinear analysis of flexible marine risers is presented. For continuous riser pipes, the hydrostatic pressure field only covers the circumferential surfaces thus hindering the buoyancy load approach. The present buoyancy load formulation is based on a vector calculus approach and essentially agrees with the well–known effective tension concept in the direct beam formulation. The gradient version of the divergence theorem (GVDT) is employed onto a continuous pipe segment subjected to circumferential hydrostatic pressure. The application of the GVDT results in the body force and cross–sectional surface traction which are regarded as the buoyancy load and boundary effect respectively. Emphasis is placed on the consequences of the boundary effect and the load definition of the buoyancy load in geometrically nonlinear analysis. The boundary effect occurs if either boundary end of the pipe system were uncapped. Further consequences of the boundary effect depends on the corresponding restraining locations and directions. Furthermore, the buoyancy load is ascertained to have weak nonlinearity hence the derivation of the augmented tangent stiffness matrix is deemed unnecessary. Finite element formulation of the corresponding external nodal load vectors is also presented using the iso-parametric beam elements for discretization.

1. Introduction

Flexible marine risers are slender pipes installed in the seawater for ocean engineering applications. Some engineering aspects of flexible marine risers include design optimization (Chen et al., 2016) and stress analysis (Tang et al., 2015). Owing to a high degree of flexibility, flexible marine risers may undergo large displacement which necessitates geometrically nonlinear analysis. Previous works (Yazdchi and Crisfield, 2002a, 2002b; Kordkheili et al., 2011; Aguiar et al., 2015; Bernitsas et al., 1985; Chucheepsakul et al., 2003; Monprapussorn et al., 2007; Athisakul et al., 2011; McNamara et al., 1988) which considered geometrical nonlinearity were based on the definition of a large displacement, small strain analysis. Here the notion of small strain is adopted according to the finite element method (Bathe, 1996; Bathe et al., 1975) and should not be confused with the consideration of the finite axial strain tensor in the works (Chucheepsakul et al., 2003; Monprapussorn et al., 2007; Athisakul et al., 2011). The aforementioned works can be generally classified into two groups according to the formulation approach. The first group (Bernitsas et al., 1985; Chucheepsakul et al., 2003; Monprapussorn et al., 2007; Athisakul et al., 2011; McNamara et al., 1988) employed Euler–Bernoulli beam theory as the formulation basis. Internal forces in the riser pipe such as axial tensile force due to top–tensioning effect were directly included in a differential beam element. Conservation of momentum was enforced based on the established differential beam element. Appropriate geometric and constitutive relations were thereby used to convert the boundary value problem from force–based into displacement–based. Since physical effects can be accounted directly through the differential beam element, such approach is termed as direct beam formulation in this paper. On the other hand, the second group (Yazdchi and Crisfield, 2002a, 2002b; Kordkheili et al., 2011; Aguiar et al., 2015) employed continuum mechanics and/or finite beam theories as the formulation basis. Conservation of momentum was enforced in a continuum mechanics setting through appropriate stress and strain definitions. In the continuum mechanics setting, mechanical forces due to physical effects would be treated as either body forces or surface tractions. Finite beam theories or their corresponding finite element implementation were thereupon employed to define kinematics and constitutive behavior of the riser pipe. As a result, the boundary value problem was converted from stress–based into displacement–based. Such approach is termed as continuum formulation with incorporated beam

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theory in this paper. In this regard, one of the key differences between both formulation approaches is how forces are incorporated. Amongst others, hydrostatic effect is essential in the ocean environment hence a proper incorporation of hydrostatic pressure force in a large displacement context is vital for riser analysis.

In the large displacement context, hydrostatic pressure force should be defined and incorporated in the geometrically nonlinear formulation as follower surface traction. For instance, augmented tangent stiffness matrices due to linearization of the external virtual work were derived in several finite element literature for pressure follower force (Hibbit, 1979; Schweizerhof and Ramm, 1984; Kreja and Cywiński, 1991; Rumpel and Schweizerhof, 2003, 2004; Hassler and Schweizerhof, 2008). For marine risers with extensive spatial domain, however, evaluation of the hydrostatic pressure force as follower surface traction is computationally cumbersome regardless of the formulation approach. In particular, the aforementioned works (Yazdchi and Crisfield, 2002a, 2002b; Kordkheili et al., 2011; Aguiar et al., 2015; Bernitsas et al., 1985; Chucheepsakul et al., 2003; Monprapussorn et al., 2007; Athisakul et al., 2011; McNamara et al., 1988) had employed an alternative method which is termed in this paper as the buoyancy load approach (Archimedes' principle) to account for the hydrostatic pressure force. Nevertheless, an important limitation of the buoyancy load approach is that the hydrostatic pressure field surrounding the body of interest must be entirely closed. For continuous riser pipes, the hydrostatic pressure field only covers the circumferential surfaces as shown in Fig. 1, and is therefore not entirely closed. Consequently, the buoyancy load approach cannot be applied for riser analysis unless the above limitation is properly addressed in the associated formulation. Such limitation had been properly overcome in the direct beam formulation through a well–known method known as the effective tension concept (Chucheepsakul et al., 2003; Sparks, 2007). The effective tension concept emerged from an intentional introduction of fictitiously equal but opposite hydrostatic pressure forces onto cross–sectional segments of the differential beam element. Superposition of all hydrostatic pressure forces acting on the differential beam element would give rise to a closed hydrostatic pressure field and axial force, thus allowing the buoyancy load approach to be valid. The effective tension at a particular position of the riser pipe is thereby defined as the sum of the internal axial tensile force due to top–tensioning effect and the axial force due to buoyancy load approach.

Whilst the validity of the buoyancy load approach had been justified in the direct beam formulation, there had not been a clear method on how the buoyancy load approach can be made valid in the continuum formulation with incorporated beam theory. A hypothetical attempt on this quest is to employ the effective tension concept from the former approach. However, the effective tension concept cannot be directly applied in the continuum formulation with incorporated beam theory.

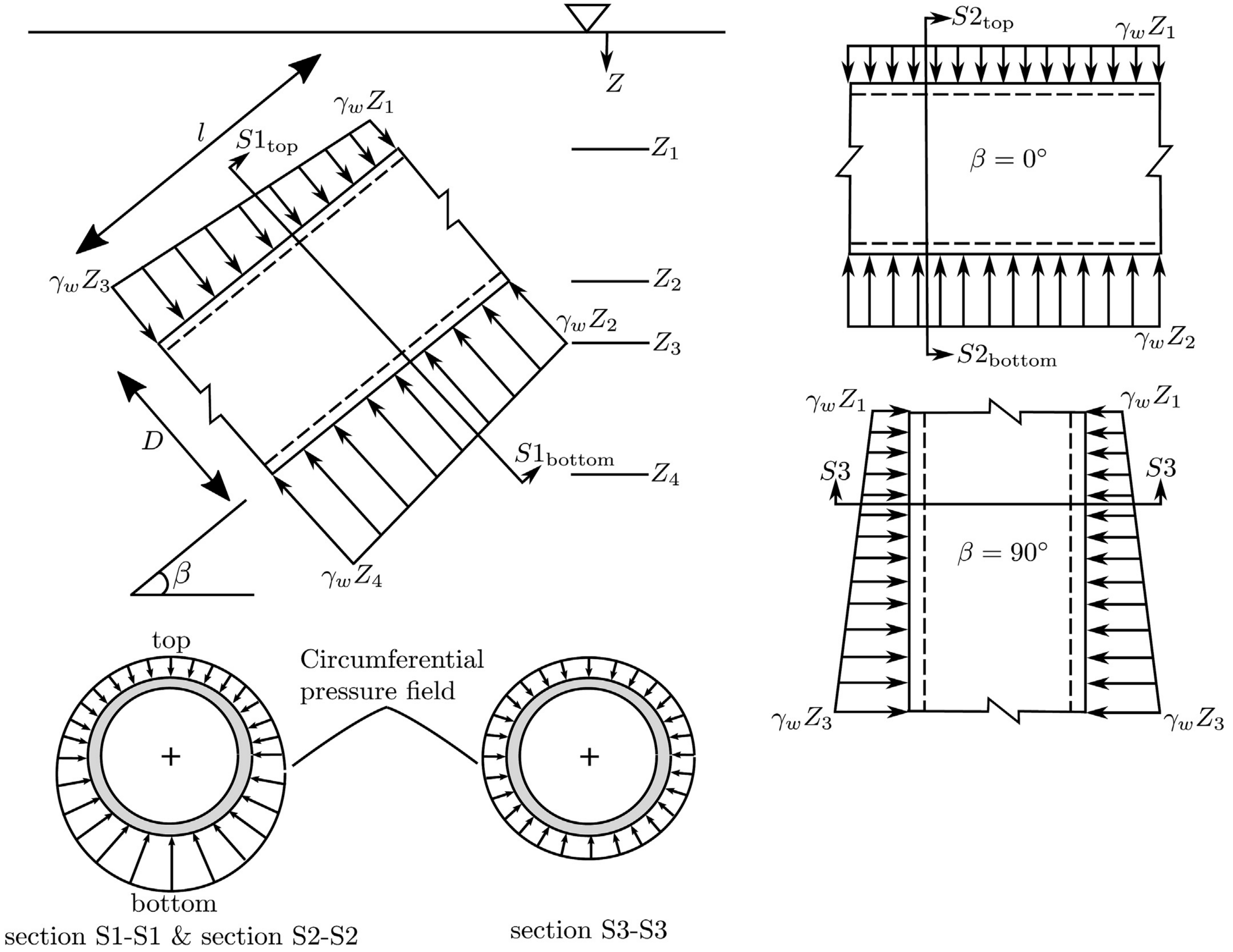

Fig. 1. Hydrostatic pressure distribution for a straight, continuous submerged pipe in incompressible fluid. Note that the cross–sectional surfaces are not subjected to hydrostatic pressure. Arrows denoting hydrostatic pressure in the slanted configuration with an acute angle β should not be confused with the follower normal force component of the buoyancy load in Yazdchi and Crisfield (Yazdchi and Crisfield, 2002a, 2002b).