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Towards straightforward use of cell-based smoothed finite element method in fluid–structure interaction

Tao He [a,b,*]

[a] Department of Civil Engineering, Shanghai Normal University, Shanghai 201418, China
[b] School of Engineering, University of Birmingham, Birmingham, B15 2TT, UK

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ABSTRACT

This paper presents a straightforward implementation of cell-based smoothed finite element method (CS-FEM) into fluid-structure interaction from the arbitrary Lagriangian-Eulerian perspective. Identical to the practice in solid mechanics, CS-FEM is directly applied to viscous stress and pressure Poisson equation in fluid problem. Minimum programming efforts are thus required to modify existing in-house codes. Following an efficient grid moving strategy, partitioned implicit coupling scheme based upon fixed-point iterations is adopted to interconnect individual fields. The proposed approach is validated against previously published data for several benchmarks. Visible improvements are exposed in predicted results along with flow-induced phenomena.

1. Introduction

Gradient smoothing is a helpful technique to stabilize nodal integration in Galerkin meshless methods (Chen et al., 2001; Yoo et al., 2004). Liu and his colleagues (Liu et al., 2007) proposed the smoothed finite element method (SFEM) by incorporation of gradient smoothing into the traditional FEM. The essential idea behind SFEM consists in modification of the compatible strain field whereby a Galerkin model may deliver some superior properties. This technique is saliently featured by its "softened" stiffness matrix which yields more accurate solution to discrete partial differential equations than FEM at the expense of easy implementation and nearly equal cost. After more than a decade of development, a family of SFEM models have been fostered on account of different smoothing domain modes. The monograph (Liu and Nguyen, 2010) and the review article (Zeng and Liu, 1007) deeply inspect SFEM's theoretical bases, deliberately highlight its advantageous traits, and vividly depict its versatility in a variety of disciplines.

Computational fluid dynamics (CFD), as it stands, probably becomes another subject of interest to SFEM practitioners. Indeed, steps are taken to study CFD related problems such as fluid-structure interaction (FSI) (Zhang et al., 2012; Yao et al., 2012; Wang et al., 2014; He, 2016). However, these scenarios simply deploy an outreach success in solid mechanics, rather than a settlement customized for the Navier-Stokes (NS) equations. Apparently, the NS equations, which constitute the principle of the majority of CFD problems, do not suit SFEM after introducing divergence theorem. For this reason, the underlying investments may be discouraged in CFD analyses. To overcome this dilemma, Jiang et al. (2018) proposed three schemes to interpolate nodal quantities of the convective acceleration for the cell-based smoothed FEM (CS-FEM) (Liu et al., 2007) for the first time. In (Jiang et al., 2018) the incompressible fluid flows within the laminar region are attempted on the Eulerian mesh.

The objective of this paper is to develop a simple smoothing treatment for FSI computation. Differing from (Jiang et al., 2018), the fluid stress tensor is partially smoothed and thus the resultant implementation is as same as that in solid mechanics. To make minimum modifications in available FE codes, the natural preference is given to the simplest CS-FEM that is initiated on four-node quadrilateral (Q4) element mesh. The characteristic-based split (CBS) scheme (Zienkiewicz et al., 1999; Nithiarasu et al., 2006) is utilized to decouple the fluid velocity and pressure. Partitioned implicit coupling strategy (He and Zhang, 2017) is preferred to interconnect individual fields due to its attractive simplicity. Furthermore, SFEM may diminish pressure sensitivity on dynamic boundaries for the fractional-step method.

The remainder of this paper is organized as follows. The basis of CS-FEM is briefly recalled in Section 2. The equations governing fluid and solid media are depicted in Sections 3 and 4. The mesh updating method is summarized in Section 5. Section 6 describes the partitioned implicit coupling algorithm. Numerical examples are investigated in Section 7 and conclusions are drawn in the final section.

* Department of Civil Engineering, Shanghai Normal University, Shanghai 201418, China.
E-mail addresses: taohe@shnu.edu.cn, txh317@bham.ac.uk.

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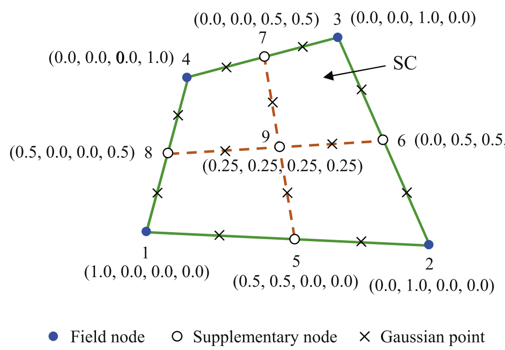

Fig. 1. Construction of SCs and shape functions in a Q4 element.

2. Brief on CS-FEM

Let us discretize a two-dimensional computational domain Ω into n_{el} Q4 elements exactly as in the standard FEM such that $\Omega = \overline{\Omega}_1 \cup \overline{\Omega}_2 \cup \cdots \cup \overline{\Omega}_{n_{el}}$ and $\overline{\Omega}_i \cap \overline{\Omega}_j = \varnothing$ $(i \neq j)$. A Q4 element is further subdivided into a set of complementary smoothing cells (SCs), namely $\overline{\Omega}_i = \tilde{\Omega}_i^1 \cup \tilde{\Omega}_i^2 \cup \cdots \cup \tilde{\Omega}_i^{nsc}$ where *nsc* is the number of SCs within the element. The gradient of a field variable q smoothing at a point $\mathbf{x}_c$ within an SC is approximated in the form of

$$\tilde{\nabla} q(\mathbf{x}_c) = \int_{\tilde{\Omega}} \nabla q(\mathbf{x}) \Phi(\mathbf{x} - \mathbf{x}_c) \mathrm{d}\Omega, \tag{1}$$

where ∇ means the gradient operator, $\tilde{\Omega}$ designates the SC and the Heaviside-type kernel Φ fulfills (Yoo et al., 2004)

$$\Phi \geqslant 0 \quad \text{and} \quad \int_{\tilde{\Omega}} \Phi \mathrm{d}\Omega = 1. \tag{2}$$

Applying Gauss theorem into the right-hand side of Eq. (1) yields

$$\tilde{\nabla} q(\mathbf{x}_c) = \int_{\tilde{\Gamma}} q(\mathbf{x}) \mathbf{n}(\mathbf{x}) \Phi(\mathbf{x} - \mathbf{x}_c) \mathrm{d}\Gamma - \int_{\tilde{\Omega}} q(\mathbf{x}) \nabla \Phi(\mathbf{x} - \mathbf{x}_c) \mathrm{d}\Omega, \tag{3}$$

where $\tilde{\Gamma}$ is the boundary of $\tilde{\Omega}$ and $\mathbf{n}$ is the unit outward normal of $\tilde{\Gamma}$. The smoothing kernel Φ is given by

$$\Phi(\mathbf{x} - \mathbf{x}_c) = \begin{cases} \dfrac{1}{A_c} & \mathbf{x} \in \tilde{\Omega}, \\ 0 & \mathbf{x} \notin \tilde{\Omega}, \end{cases} \tag{4}$$

where $A_c = \int_{\tilde{\Omega}} \mathrm{d}\Omega$ is the area of the SC. Substituting Eq. (4) into Eq. (3), we have

$$\tilde{\nabla} q(\mathbf{x}_c) = \int_{\tilde{\Gamma}} q(\mathbf{x}) \mathbf{n}(\mathbf{x}) \Phi(\mathbf{x} - \mathbf{x}_c) \mathrm{d}\Gamma = \frac{1}{A_c} \int_{\tilde{\Gamma}} q(\mathbf{x}) \mathbf{n}(\mathbf{x}) \mathrm{d}\Gamma, \tag{5}$$

where the gradient of a constant automatically vanishes.

The Galerkin procedure gives the following approximation of q

$$q = N_I \underline{q}_I, \tag{6}$$

where N_I is the shape function at node I, the underline indicates a nodal quantity and Einstein's summation is applied. With the aid of Eq. (6), one can immediately rewrite Eq. (5) as

$$\tilde{\nabla} q(\mathbf{x}_c) = (\tilde{\nabla} N_I(\mathbf{x}_c)) \underline{q}_I = \left(\frac{1}{A_c} \int_{\tilde{\Gamma}} N_I(\mathbf{x}) \mathbf{n}(\mathbf{x}) \mathrm{d}\Gamma \right) \underline{q}_I. \tag{7}$$

Since one-point Gaussian quadrature is sufficiently accurate for line integral along each segment of $\tilde{\Gamma}$, the item enclosed within external brackets on the right hand side of Eq. (7) can be transformed to its algebraic form

$$\tilde{\nabla} N_I(\mathbf{x}_c) = \frac{1}{A_c} \sum_{i=1}^{4} N_I(\mathbf{x}_i^{gp}) \mathbf{n}(\mathbf{x}_i^{gp}) l_i, \tag{8}$$

where $\mathbf{x}_i^{gp}$ is the Gaussian point of the boundary segment $\tilde{\Gamma}_i$ and l_i is the length of the i-th segment.

As of now, no coordinate transformation is involved and only shape functions are invoked to calculate the smoothed gradients. A Q4 element is partitioned into four quadrilateral SCs, relying on the stability condition (Liu et al., 2007). The construction of shape functions for CS-FEM is illustrated in Fig. 1. Of total nine nodes, extra five nodes are generated to compute the smoothed shape functions by simply averaging those values at four corners (Liu et al., 2007; Dai and Liu, 2007). Therefore, no additional degrees of freedom are introduced into CS-FEM.

3. Fluid problem

3.1. Governing equations

Let $\Omega^f \subset \mathbb{R}^2$ and $(0,\ T)$ be the fluid and temporal domains, respectively. Ω^f is bounded by Γ^f which is decomposed into three complementary subsets, i.e., Dirichlet boundary Γ_d^f, Neumann boundary Γ_n^f and fluid-structure interface Σ. The spatial and temporal coordinates are denoted as $\mathbf{x}$ and t. The isothermal incompressible viscous fluid flows on a moving mesh are dominated by the ALE formulation of the NS equations as follows

$$\nabla \cdot \mathbf{u} = 0 \quad \text{on} \quad \Omega^f \times (0,\ T), \tag{9}$$

$$\rho^f \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{c} \cdot \nabla \mathbf{u} - \mathbf{f}^f \right) - \nabla \cdot \boldsymbol{\sigma}^f = 0 \quad \text{on} \quad \Omega^f \times (0,\ T), \tag{10}$$

where $\mathbf{u}$ is the fluid velocity, ρ^f is the fluid density, $\mathbf{c} = \mathbf{u} - \mathbf{w}$ is the convective velocity, $\mathbf{w}$ is the mesh velocity, $\mathbf{f}^f$ is the body force, and $\boldsymbol{\sigma}^f$ is the fluid stress.

The constitutive equation for Newtonian fluid reads as

$$\boldsymbol{\sigma}^f = -p\mathbf{I} + 2\mu\varepsilon \quad \text{and} \quad \varepsilon = \frac{1}{2}(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \quad \text{on} \quad \Omega^f \times (0,\ T), \tag{11}$$

where p is the fluid pressure, $\mathbf{I}$ denotes the identity tensor, μ is the dynamic viscosity, ε indicates the rate-of-strain tensor and superscript T means transpose.

The fluid problem is completed by prescribing initial and boundary conditions below

$$\mathbf{u}(\mathbf{x},\ 0) = \mathbf{u}^0, \quad p(\mathbf{x},\ 0) = p^0 \quad \text{on} \quad \Omega_0^f, \tag{12}$$

$$\mathbf{u} = \mathbf{g}^f \quad \text{on} \quad \Gamma_d^f, \quad \boldsymbol{\sigma}^f \cdot \mathbf{n}^f = \mathbf{h}^f \quad \text{on} \quad \Gamma_n^f,$$

where $\mathbf{n}^f$ is the unit outward normal of Γ_n^f. The interface coupling conditions will be presented in a separate subsection.

We define the following dimensionless scales

$$\hat{\mathbf{x}} = \frac{\mathbf{x}}{L}, \hat{t} = \frac{tU}{L}, \hat{\mathbf{u}} = \frac{\mathbf{u}}{U}, \hat{\mathbf{c}} = \frac{\mathbf{c}}{U}, \hat{p} = \frac{p}{\rho^f U^2}, \hat{\mathbf{f}}^f = \frac{\mathbf{f}^f L}{U^2}$$

where L is the characteristic length and U the free-stream velocity. By employing these variables and dropping all hats, the dimensionless ALE-NS equations are cast as follows

$$\nabla \cdot \mathbf{u} = 0, \tag{13}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{c} \cdot \nabla \mathbf{u} - \nabla \cdot \boldsymbol{\sigma}^f - \mathbf{f}^f = 0, \tag{14}$$

along with the constitutive relation