Ocean Engineering 157 (2018) 387–400

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Nonlinear real time prediction of ocean surface waves

Nikolai Köllisch [a,*], Jasper Behrendt [a], Marco Klein [b], Norbert Hoffmann [a,c]

[a] Technische Universität Hamburg, Mechanical Engineering, 21073, Hamburg, Germany
[b] Technische Universität Hamburg, Institute for Ship Structural Design and Analysis, 21073, Hamburg, Germany
[c] Imperial College London, Mechanical Engineering, London, SW7 2AZ, United Kingdom

ARTICLE INFO

Keywords:
Wave prediction
High order spectral method
Nonlinear waves

ABSTRACT

A highly efficient numerical procedure for the computer based prediction of nonlinear deep water ocean surface waves is presented. To reconstruct consistent initial conditions from measured surface elevation data, the approach employs a combination of a high order spectral method with Krylov subspace techniques. Aiming at prediction horizons of a few minutes, the method allows real-time prediction of sea states for domains of several square kilometres. Taking the nonlinearity of the wave evolution equations into account leads to a substantial increase in accuracy of the prediction at only moderate additional cost.

1. Introduction

The dynamics of ocean waves is a highly multi-faceted topic. Recently, with improved observational and computational resources now at hand, an increased interest has emerged in exploring the potential of computer simulation based phase resolved wave prediction. Predictions of ocean waves with a temporal horizon of a few minutes could have a large impact on a number of applications in ocean engineering.

Methods to achieve short term phase resolving wave forecasting may be based on time series analysis (Fusco and Ringwood, 2010; Birkholz et al., 2015; Faller et al., 2013; Kosleck) or on solving the physical evolution equations of ocean wave using computer simulations. In the latter case the prediction process consists of two stages: the extraction of initial conditions from measurement data (sometimes also called the assimilation process), and the forward integration in time. The methods can be further subdivided, depending if they rely on linear or nonlinear models for the wave evolution. Examples of linear predictions are given in Kosleck, Ruban (2016), Naaijen and Wijaya (2014) and Wijaya et al. (2015). The popularity of the use of linear water wave theory is mostly due to its computational efficiency. The demand of wave predictions due to safety issues has recently caused an enhanced interest in nonlinear models. In van Groesen et al. (2017) for example a nonlinear simulation is used for the simulation of extreme events. However, obtaining results faster than real time is challenging when using nonlinear simulation methods (Faller et al., 2013; Blondel et al., 2010; Blondel-Couprie et al., 2013). The long computation times required for such models is mostly caused by the data assimilation procedure (Blondel-Couprie et al., 2013) rather than by the forward integration in time.

In this work a nonlinear model based prediction tool for deep water surface gravity waves will be presented. First a discussion of the prediction domain which has some relevance for the choice of the data acquisition arrangement is given. Then the simulation method is described. The physical model is restricted to potential theory, for which the equations for ocean surface waves are solved by applying a high order spectral method (HOS), which was developed independently by West et al. (1987) and Dommermuth and Yue (1987).

Since achieving real time simulation is one of the biggest challenges, a new efficient approach for the data assimilation will be presented in section 2.3. Initial surface elevation data, as it may arise from measurements, and the nonlinear wave evolution equations are used to determine consistent initial conditions for the subsequent time-integration. The process yields initial conditions for both the initial surface height as well as for initial surface potentials that are fully consistent with the nonlinear wave evolution. By this the assimilation process copes with errors and incompleteness in initial measurement data. Compared to other approaches to the assimilation process, the approach aims at high numerical efficiency, necessary to reach real time simulations based on initial observational data.

After all methods have been introduced, the approach is applied to two test cases.

* Corresponding author.
E-mail address: dynamics.group@tuhh.de (N. Köllisch).

https://doi.org/10.1016/j.oceaneng.2018.03.048
Received 15 January 2017; Received in revised form 28 February 2018; Accepted 14 March 2018
Available online 5 April 2018

2. Methods

2.1. Prediction region

For subsequent use, we define the prediction region as the spatio-temporal domain for which prediction can be accomplished for given measurement data. In other words, when the sea state is known for certain ranges in space and time, the prediction region indicates which spatio-temporal domain can be forecasted. There has been a lot of work carried out related to the theoretical determination of the prediction region, Wu (2004), Blondel-Couprie (2009) and Kosleck to name only a few, and a good overview of the topic is given in Naaijen et al. (2014). The aim of this section is not to discuss the details of the determination of the prediction region, but rather to identify which kind of measurements should be employed for wave forecasting in general to yield a larger rather than a smaller prediction region.

The main component determining the size of the prediction region is the group velocity (Naaijen et al., 2014), which is under the assumption of small wave steepness determined using linear theory (Clauss et al., 1988):

$$c_g = \frac{\partial\omega}{\partial k} \approx 0.5\sqrt{\left(\frac{g}{k}\tanh kd\right)}\left(1+\frac{2kd}{\sinh 2kd}\right) \quad (1)$$

Here ω is the angular frequency of the wave component and k is the corresponding wave number ($2\pi/\lambda$), which is linked to the frequency by the linear dispersion relation

$$\omega = \sqrt{gk\cdot\tanh(kd)}, \quad (2)$$

with λ the wave length, g the gravitational constant and d the water depth.

The key factors determining the spatio-temporal prediction regions are the group velocities contained in the wave state at hand, since, at least from a perspective of linear wave propagation, the group velocities determine the propagation of the energy content of the wave state. So, for example, if a measurement is taken at a single given point for a certain interval in time, a wedge-like shape results for the prediction region. It is bounded by the wave components with maximum and minimum group velocity involved, as shown in Fig. 1. If there is more than one measurement point, the prediction region expands (Wu, 2004), as shown in Fig. 2. This suggests that a simple way to increase the prediction region is to collect data at more than a single spatial position, or ideally over a whole spatial range. Of course the prediction region also increases when the measurement period of an individual device at a single position is increased. However, taking measurements at more spatial positions into account yields an additional benefit, denoted as the extended prediction region in Fig. 2.

To visualise this further, and also to take into account that the identification of a maximum and minimum group velocity within a real sea states is always somehow ad-hoc, a more general definition of the prediction error can be employed. For example Wu (2004) has introduced the following quantitative energy based measure for the prediction error,

$$\varepsilon = \sqrt{1-\frac{\int_{\omega_l}^{\omega_h} S(\omega)d\omega}{\int_0^\infty S(\omega)d\omega}}, \quad (3)$$

where $S(\omega)$ denotes the power spectral density of the wave state under consideration, i.e. $S(\omega)$ describes how the wave energy is distributed in frequency space. Here the two frequencies ω_h and ω_l are taken as the highest and the lowest relevant energy containing wave frequencies of the spectrum, relevant for the prediction purpose. In contrast to the qualitative definition of predictability as used above, this definition introduces a scale on the unit interval.

Comparing prediction regions or prediction errors for purely

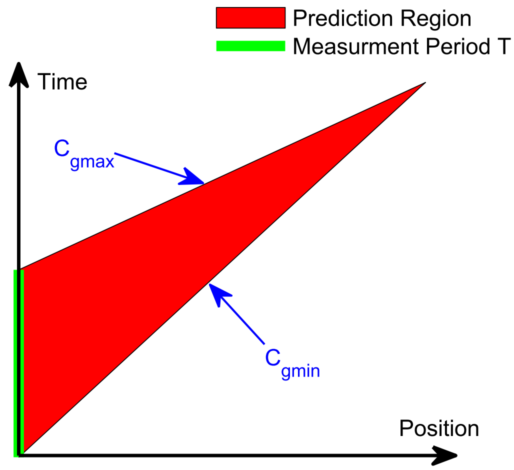

Fig. 1. Prediction region obtained for the case of a single measurement device at a fixed position, resulting in data for a given time interval.

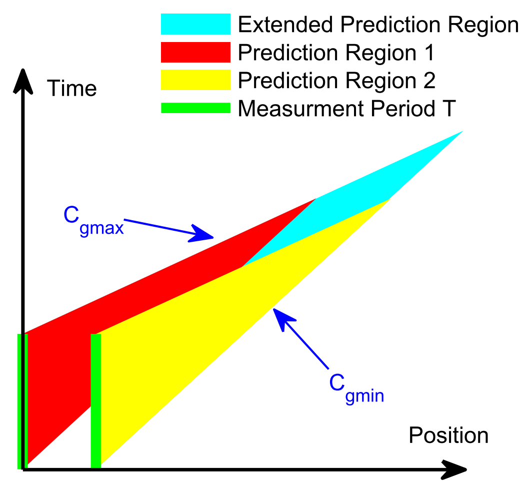

Fig. 2. Prediction region obtained for the case of two measurement devices at adjacent locations.

temporal and purely spatial domains of initial data by performing nonlinear simulations, see Fig. 3, suggests that initial spatial measurements might form a good starting point. Although both approaches could in principle also be combined, and our assimilation procedure described below to some extent also takes into account a certain temporal interval by including time-derivatives, in the following we focus on initial spatial measurement data.

2.2. Simulation method

In this study the prediction of wave states is to be accomplished through numerical simulation of the wave field starting with spatial initial conditions. As widely done, we restrict the approach to potential flow, neglecting viscosity and vorticity. The flow field is given by the scalar velocity potential Φ, satisfying the Laplace equation,

$$\Delta\Phi = \nabla\cdot\nabla\Phi = 0. \quad (4)$$

The velocity field can be obtained by

$$\mathbf{u} = \nabla\Phi. \quad (5)$$

The kinematic and dynamic boundary conditions for the free surface