



## Effects of vertical porous barrier on progressive waves in a two layered fluid

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## ABSTRACT

Two dimensional scattering of linear water wave by thin vertical permeable plate in a two-layer fluid with free surface is considered. The permeable barrier is completely submerged in the upper layer of finite depth over a layer whose depth is either infinite or finite. For discontinuity in the potential across the plate, we use Green's integral theorem to formulate the problem in terms of a hypersingular integral equation. A collocation method using a finite series of Chebyshev polynomials of second kind have been introduced to get the unknown difference potential numerically. The reflection and transmission coefficients for surface and internal modes are computed as an integral involving difference potentials. The proportion of reflected and transmitted energies of both the wavenumbers are calculated. Moreover, amount of energy dissipated due to the presence of permeable plate in the upper layer are obtained. Numerical results are calculated and depicted graphically against the wavenumber for various non-dimensionalized parameters.

## 1. Introduction

Problems involving scattering of water waves in a fluid by an obstacle are widely investigated by many researchers over the years due to its possible applications. The scientists often used a technique, namely linearized water wave theory, where the amplitude of waves are assumed to be very small compared to other length scales in the problem. Recently, interest has been extended to bodies which are immersed in two-layer fluids having a different density. One reason for this is the suggestion by Friis et al. (1991) that an underwater pipe bridge might be built across one of the Norwegian fjords, which typically consists of a layer of fresh water of finite depth on top of a very deep body of salt water.

The propagation of water waves in a two-layer fluid in absence of any obstacle was described by Stokes (1847) long back, till now many researchers worked on study of water wave propagation in multilayered fluids with an interface (see, Kassem (1982, 1986), Chakrabarti and Mandal (1983), Mandal and Chakrabarti (1986), Rhodes-Robinson (1994)). Kassem (1982, 1986) considered the problems dealing with the generation of internal waves at the surface separating two fluids involves the consideration of different types of singularities in one of the two finite depth fluids neglecting surface tension at the interface. Chakrabarti and Mandal (1983), Mandal and Chakrabarti (1986), Rhodes-Robinson (1994) studied the effects of singularities in either of the layers in an

infinite depth fluid by methods similar to those which are used for the corresponding one fluid problems. In these investigations interests have been centered on the determination of the potentials for two dimensional wave sources or multipoles and axisymmetric multipoles. Linton and McIver (1995) developed the general theory for two dimensional motion in a two layer fluid in which the lower layer of heavier density extends infinitely downwards and the upper layer of lower density has a free surface. They studied the problem of wave scattering by a horizontal circular cylinder in either the upper or lower layer by using a method based on multipole expansions. Das and Mandal (2006) extended the problem of Linton and McIver (1995) where the upper layer is considered to be bounded by a thin ice sheet.

In last few decades there is a considerable development in the field of offshore constructions. A submerged structure such as breakwaters plays significant role in reducing the wave loads acting on various marine facilities. A significant reason for the construction of submerged breakwaters is that free exchange of water mass through the structures is usually necessary so that the water in the sheltered region can be kept circulating and, therefore, prevented from pollution. Scattering of surface waves by a thin vertical barrier submerged in a single layer fluid is considered by Dean (1948), Ursell (1950), Evans (1970) and others for deep water, wherein the barrier is either fully submerged and extends infinitely downwards or partially immersed or submerged in an infinitely deep water. McIver (1985) exercised the scattering of waves by a pair of

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surface-piercing, vertical barriers using a method based on eigenfunction expansions and described a theory to determine the resonant frequencies at which the reflection and transmission coefficient vanish. Porter and Evans (1995) employed method of solving water wave scattering by barriers in a finite depth water based on a Galerkin approximation. A theoretical study for water wave interaction with a floating body of arbitrary shape in a two layered fluid of finite depth was established by Ten and Kashiwagi (Ten and Kashiwagi, 2004). They proposed an efficient numerical method of evaluating the velocity potentials by boundary integral equation method. The scattering of surface water waves by a semi-infinite floating membrane due to abrupt change in bottom topography is analyzed using expansion formulae by Karmakar and Sahoo (2008).

Barriers considered in the literature described above are impermeable. But in case of a permeable structure, it has the ability of energy dissipation, so that transmission of energy through the barrier are reduced, which is of great interest in many ocean engineering applications. Studies related to water wave scattering by porous barriers have been carried out by a list of researchers. A porous wavemaker theory to study the generation of water waves by the harmonic oscillation of a thin porous plate, extended from the free surface to the channel bottom in water of finite depth have been introduced by Chwang (1983). Water wave diffraction by porous breakwaters have been studied by Yu (1995). Chwang (Chwang and Chan, 1998) gave a general review on the interaction between porous media and wave motion. Their studies focused on the effect of a porous structure on incoming wave trains and the movement of waves past a plate with regular gaps in it. Using the method of eigenfunction expansion in conjunction with least square approximations, Lee and Chwang (2000) studied water wave scattering and radiation problems. They have considered different configurations of the porous barrier. Sahoo et al. (Sahoo et al., 2000) extended the work of Lee and Chwang (2000) by studying oblique wave scattering by a porous barrier and analyzed the variations of reflection coefficients, hydrodynamic pressure differences along the two sides of the barriers, and surface elevations for different parameters. Manam and Sahoo (2005) analyzed the scattering of oblique water wave by a fully extended porous barrier in a two layer fluid having a free surface and an interface. They used a modified orthogonal relation to solve the problem using eigenfunction expansion method and derived the explicit forms of the reflection and transmission coefficients. The scattering of surface wave by a thin permeable flexible barrier in a two layered finite depth fluid with free surface where the barrier is considered to be fixed at two ends have been studied by Kumar et al. (Kumar et al., 2007) and Behera et al. (Behera et al., 2013). A hydrodynamic model of perforated or slotted structures have been proposed by Molin (2011) and different cases of application are presented: two and three-dimensional cylinders, wave absorbers consisting in perforated plates, plates and disks and water entry of perforated wedges. Behera et al. (Behera et al., 2015) investigated the phenomenon of the oblique wave scattering by a rectangular porous structure in two-layer finite depth fluid. The associated mathematical problem is solved by eigenfunction expansion method.

A variety of methods such as finite element method, eigenfunction expansion method and boundary integral equation method have been

used extensively to study water wave scattering by barriers. These methods are so efficient that results of many experimental works are compared with theoretical works with success. Cho and Kim (2008) studied the oblique wave interaction with porous plates of different configurations using matched eigenfunction expansion method and boundary element method. They have correlated the theoretical results through experiments with horizontal/inclined/dual porous plates. In these methods the inevitable singularities at the end points of the barrier have not been considered. Martin (1991) developed a method for determining the behavior of the solutions to a hypersingular integral equation near the end points of the interval of integration. [Parsons and Martin (1992, 1994)] have studied the problem of water wave scattering by a vertical plate and a curved plate submerged in a single-fluid using hypersingular integral equation over the plate for difference potential. This approach has several advantages. For instance, the radiation condition is automatically satisfied by the choice of Green's function. Similarly, the behavior of difference potential at each edge of the plate, where there are square root zeros, can be easily enforced. Moreover, the method is applicable to curved plates as well as flat plates. In fact, apart from some simple quadratures, the only approximation required is that of a bounded function defined on a finite interval. This can be done by choosing an appropriate set of orthogonal polynomials, namely Chebyshev polynomials of the second kind, and then using a collocation method on the governing integral equation. Using hypersingular integral equation approach the problem of wave scattering by a porous plate in a single layered fluid was studied by Gayen and Mondal (2014). Mandal et al. (1995) considered two superposed fluids of infinite depth and a thin vertical barrier of infinite length submerged completely in the lower layer. Dhillon et al. (2014) considered the problem of wave scattering by a vertical partially immersed barrier in a fluid consisting of a layer of finite depth bounded above by a free surface and below by an infinite layer of fluid of density greater than the upper layer. Sarkar et al. (2014) developed a mathematical model to investigate the hydrodynamic behavior of a wave energy farm consisting of a large number of oscillating wave surge converters in arbitrary configuration with oblique wave incidence. The application of Green's integral theorem yields hypersingular integral equations which are then solved by using the Chebyshev polynomials of the second kind.

In the present paper, we consider the scattering of water waves by a permeable thin vertical submerged plate in the upper layer of two superposed fluids. The upper layer being of finite depth and having a free surface while the lower layer extending finitely as well as infinitely downwards. In such a two layer fluid, wave can propagate with two wave modes. When the density ratio of the fluid is very small, one can interpret these different modes very easily. For an arbitrary but stable density ratio, there is a possibility that some of the energy may transferred from one mode to another due to an obstacle in the wave field. By suitable application of Green's integral theorem in two fluid regions, the problem is formulated as a hypersingular integral equation for difference of potentials across the barrier. A collocation method involving Chebyshev polynomials of second kind is applied to solve the hypersingular integral equation. This reduces the equation to a system of linear equations which can be solved by standard methods. The solutions are used to get the reflection and transmission coefficients for incident waves of both the modes. The energy conservation relations due to the presence of porous barrier are obtained in two dimensional context. The numerical values for the proportion of energy reflected, transmitted and dissipated for incident waves, two different modes are computed and depicted graphically to demonstrate the effect of depth ratio and porosity of the thin plate. The energy conservation relations in a single layer fluid are calculated and analyzed numerically to check the correctness of the numerical results presented here.

## 2. Formulation of the problem

We consider irrotational motion in a two layered inviscid, incom-

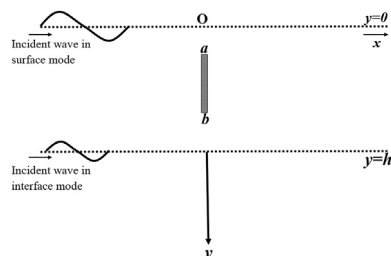


Fig. 1. Schematic diagram.

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