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A comparison between two switching policies for two-unit standby system

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ABSTRACT

The standby has been widely applied to improve the reliability of system. And the standby unit is usually activated only when the active units fail under the common switching policy. But it would not always make the system most reliable. In this paper, based on a two-unit standby system without repair, we introduce the active switching policy in which the standby unit is activated at either a pre-fixed time or the failure time of active unit. Considering the perfect and imperfect switching, the survival function and mean time to failure of system are derived using the general time-to-failure distribution under the active and common switching policy, respectively. Further, if the lifetimes of units follow the exponential distribution, the cases where the active switching policy is superior are specified clearly. For the Weibull distribution, an application example is presented and it demonstrates that the active switching policy sometimes is still superior. Besides, the essence of the active switching policy and the reason why it is more effective are simply discussed. The study proves that it is likely to make the system more reliable by adopting other switching policy rather than the common one.

1. Introduction

To improve the reliability and availability of system, the common design technique is standby, which has been widely accepted in applications. In standby system, one or more units are in active state and other standby units are in inactive state. When there are failures among the active units, the standby units are activated to replace the failed ones. Generally, there are three types in standby: hot, warm and cold standby. In hot standby, the inactive units undergo the same operational environment as when they are in active state. For cold standby, the standby units would not fail in inactive state. And the intermediate type between the cold and hot standby is just the warm standby, which means that an inactive unit has a failure rate between that for the cold and hot standby.

Extensive literature exists with regard to the reliability of standby system. The basic problem is how to assess the reliability of standby system. Many methods are proposed, such as the Markov model [1], counting process [2], analytical methods [3], equivalent age [4], Monte Carlo simulation [5] and Bayes theory [6]. Another aspect is the redundancy allocation problem concerning the choice of redundancy strategies and the selection of redundancy level [7,8]. Besides, the repair is also considered for the standby system [9]. It causes the problem of maintenance policy and surveillance testing [10–12].

Notice that all the above study is based on the assumption that the standby units are switched to the active state only when the active units fail. In this paper, we call this way of switching the common switching policy. Intuitively, the common switching policy would improve the reliability of standby system. But in engineering, we have encountered the other switching policy, illustrated using the two-unit standby system in Fig. 1. For the standby system with highly reliable units, it may cost long time for the failure of active unit. Then, after a period of time, the practitioners activate the standby unit and switch the active unit to the standby state before the active unit fails. If there is still no failure of the active unit after the switching, the active and standby units are switched again. Of course, if the active unit fails, the unfailed standby unit would still be activated immediately. This rather novel switching policy has been performed in engineering, such as the standby sensor system.

For the comparison between the common and other switching policies, Li et al. [13] studied this problem by considering the k-out-of-n: G system with a single standby unit. For this configuration, under the common switching policy, the standby unit is activated only at the $(n-k+1)$th failure of active units. But it is proposed that the standby unit could be switched to the active state at the $(n-k-m+1)$th failure for $m=0, \ldots, n-k$. Further, it is proved that the proposed switching policy indeed makes the system more reliable than that under the common one in several scenarios [13]. But the switching time points in [13] are still the failure times of active units rather than arbitrary times.

The novel switching policy in engineering and the research in [13] motivate us to study whether there is more effective

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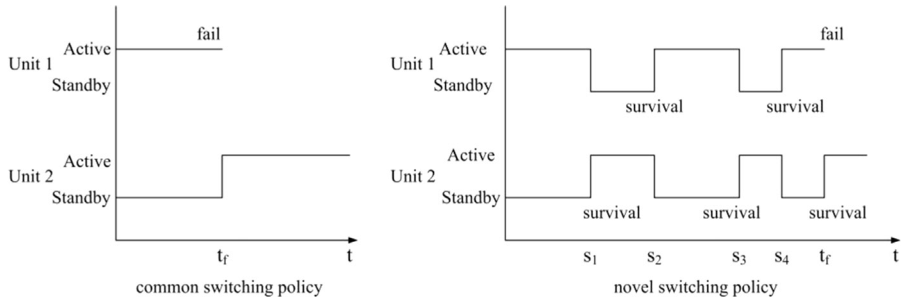

Fig. 1. The illustration of the common and novel switching policy.

switching policy than the common one generally. In this paper, we introduce the active switching policy in addition to the common switching policy based on a two-unit standby system without repair. For the two-unit standby system, one unit is in active state and the other one is in standby state initially. Under the common switching policy, the standby unit is activated only when the active unit fails. And the active switching policy here is elicited from the novel switching policy mentioned above. Under the active switching policy, if the active unit fails, the unfailed standby unit is activated in the meanwhile. In addition, by setting a fixed time point s, the unfailed standby unit is also switched to the active state at time s if the active unit survives until s. The aim of this paper is to compare the two switching policies and find the optimal s if the active switching policy is superior.

The rest of the paper is organized as follows. A general model is introduced for lifetime distributions under different environments in Section 2. In Section 3, according to this general model, the survival function and mean time to failure (MTTF) of the standby system are derived under the active and common switching policy, respectively. Next, the two switching policies are compared in exponential and Weibull distributions cases in Section 4. In Section 5, an application example is provided and the results are discussed. Finally, the paper is concluded in Section 6.

2. Lifetime distributions under different environments

To describe the lifetimes under different environments, Cha et al. [14] proposed a general model according to the ideas in accelerated life tests and the concept of equivalent age. In this section, this general model is introduced briefly.

Denote random variable X the lifetime of a unit in the usual level of environment and $F(t)$, $R(t)$ the cumulative distribution function (CDF) and survival function of X, respectively. Also let random variable X_a represent the lifetime of a unit in the accelerated level of environment and $F_a(t)$, $R_a(t)$ be the CDF and survival function of X_a. For any $t \ge 0$, it is natural that $F(t) \le F_a(t)$ implying $F_a(t) = F(\rho(t))$, where $\rho(t) \ge t \ge 0$ and $\rho(0) = 0$. Further, let random variable X_m be the lifetime of a unit in the milder level of environment and $F_m(t)$, $R_m(t)$ be the corresponding CDF and survival function. Similarly, for any $t \ge 0$, we have $F_m(t) \le F(t)$ meaning $F_m(t) = F(w(t))$, where $0 \le w(t) \le t$ and $w(0) = 0$. The general model here could associate all the lifetime distributions under different environments with the distribution under the usual level of environment easily.

The warm standby unit undergoes the milder environment than that of the active unit. If the warm standby unit survives until u and is activated at u, the CDF of the residual lifetime X^r of standby unit is

$$P(X^r \le t | X_m \ge u) = \frac{F(t+w(u))}{R_m(u)} = P(X^r \le t | X \ge w(u)) = \frac{F(t+w(u))}{R(w(u))}, \quad \forall t \ge 0. \tag{1}$$

3. System performance

In this section, the survival function and MTTF of the two-unit warm standby system are derived under the active and common switching policy, respectively.

3.1. Assumptions

1. The unit 1 is operated and unit 2 is in the warm standby state initially.
2. The lifetimes of units follow the general time-to-failure distribution.
3. The warm standby system is not repairable during the mission.
4. The switching from the standby to active state is instantaneous [15].

3.2. Perfect switching

In this subsection, we consider that the switching is failure free. Let $F_i(t)$, $f_i(t)$ and $R_i(t)$ be the CDF, probability density function (PDF) and survival function of the unit $i (i=1,2)$ in the active state, respectively. Under the common switching policy, if the standby system survives at time t, either the unit 1 survives at time t or the unit 2 survives the remaining time after the unit 1 fails before time t. Then the survival function of system is

$$R_c(t) = \int_t^{+\infty} f_1(u)du + \int_0^t f_1(u) \cdot R_{w2}(u) \cdot \frac{R_2(t-u+w_2(u))}{R_2(w_2(u))} du, \tag{2}$$

where $R_{w2}(t)$ is the survival function of unit 2 in warm standby state and $R_{w2}(t) = R_2(w_2(t))$. Further, the MTTF of the standby system here is $ET_c = \int_0^{+\infty} R_c(t)dt$.

For the active switching policy, the fixed switching time point s is specified. If $t \le s$, the survival function of system is just $R_s(t) = R_c(t)$. Otherwise, when the system survives at time t, it consists of the following four exclusive events, illustrated in Fig. 2.

Event 1: The unit 1 fails before time s and the unit 2 survives until time t. In this case, the survival function is

$$P_1(t,s) = \int_0^s f_1(u) \cdot R_{w2}(u) \cdot \frac{R_2(t-u+w_2(u))}{R_2(w_2(u))} du. \tag{3}$$

Event 2: The unit 2 fails before time s and the unit 1 survives until time t. Here, the survival function is

$$P_2(t,s) = R_1(t) \cdot F_{w2}(s) = R_1(t) \cdot F_2(w_2(s)). \tag{4}$$

Event 3: Both the two units are not failed at time s. Then the unit 2 is activated at s and survives the remaining time. And the survival function is

$$P_3(t,s) = R_1(s) \cdot R_{w2}(s) \cdot \frac{R_2(t-s+w_2(s))}{R_2(w_2(s))}. \tag{5}$$

Event 4: Both the two units are not failed at time s. Then, the unit 2 is activated and the unit 1 is switched to the standby state. But the unit 2 fails before time t. So the unit 1 is activated again