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Nonlinear vibrations of offshore floating structures moored by cables

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A R T I C L E I N F O

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A B S T R A C T

In this study, a numerical model is presented for the nonlinear vibrational analysis in the symmetrical plane of the rectangular offshore floating structures moored by cables. The upper end of each mooring cable is connected to the floating structure and the other end is fixed to the sea bed. The nonlinear equations of motions of the mooring cables are derived by using nonlinear cable elements that are formulated based on the extended Hamilton principle. The floating platform is modeled as a rigid body with three degrees of freedom. The forces applied on the floating structure and cables are analyzed, formulated and expressed in details. The connection conditions between the floating structure and mooring cables are introduced to formulate the equations of motions of the system as a whole. The vibrations of the floating structure under horizontal sinusoidal excitation are analyzed numerically. The influence of different sag-to-span ratios or inclined angles of the mooring cables, and that of different current velocities on the displacements of the floating structure and maximum tensile force in the cables are studied. The displacement amplitudes of the moored floating structure and maximum cable tensile force under different current velocities are also studied for different excitation frequencies.

1. Introduction

Large floating structures have been widely used in ocean engineering in the last few decades as they are financially economical, can be constructed quickly, and easily expanded and removed. They are one of the most environmentally friendly innovations that allow for the creation of artificial land in the sea without destroying marine habitats or polluting coastal waters (Wang et al., 2007). Moored floating structures are one of the most popular types of offshore platforms, and used to extract marine resources, such as oil, gas and minerals. These structures consist of a floating platform and mooring cables. If the floating platform is subjected to horizontal excitation, its movement can induce changes in the mooring cable geometry. Consequently, the geometric nonlinearity of the mooring cables may substantially affect the behavior of the floating platform due to their flexibility. Therefore, the accurate modeling of mooring cables is crucial for carrying out the vibrational analysis of moored floating structures.

The mooring cables were simplified as linear springs by some researches (Yamamoto et al., 1980; Sannasiraj et al., 1998; Tang et al., 2011). The slack mooring cables were modeled as linear springs to support the floating platform. The stiffness coefficients are derived from the catenary equations of the cables. After the spring constants are determined, they are added to the linear stiffness in the equations of motion of the floating platform. The advantage of this approach is that it is more convenient and efficient for numerical analysis, but the results lack accuracy because the behaviors of the cables cannot be well reflected simply by the linear springs. This kind of structure has also been modeled as a rigid mass connected to nonlinear springs (Esmailzadeh and Goodarzi, 2001; Agarwal and Jain, 2003; Umar and Datta, 2003; Rosales and Filipich, 2006). The dynamic tension from the mooring cables that acts on the floating platform is taken into consideration based on the geometry of the catenary chains and expressed with nonlinear terms in terms of the displacement, velocity and acceleration of the floating platform. The vibration of the mooring cables was analyzed by using the lumped mass model for a more accurate analysis of the moored floating structure (Huang, 1994; Masciola et al., 2012; Zhu and Yoo, 2015, 2016). With this approach, the cables are divided into sections which are connected by nodes, and equilibrium equations are directly formulated at each node. Mooring cables have also been modeled by using a bar element as proposed by Garrett (1982, 2005) or a bar element combined with an updated Lagrangian formulation (Gutiérrez-Romero et al., 2016). The two-node catenary cable element was formulated based on the exact analytical geometry of elastic catenary and the tangent stiffness matrix was derived (O'Brien and Francis, 1964; Jayaraman and Knudson, 1981; Yang and Tsay, 2007). The finite element method was used to model the mooring cables based on the principle of minimum energy which

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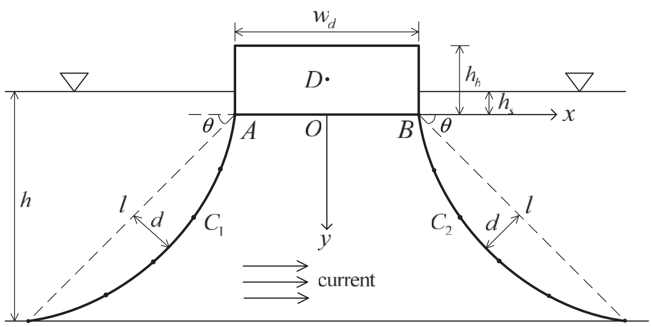

Fig. 1. Configuration of the two-dimensional floating system.

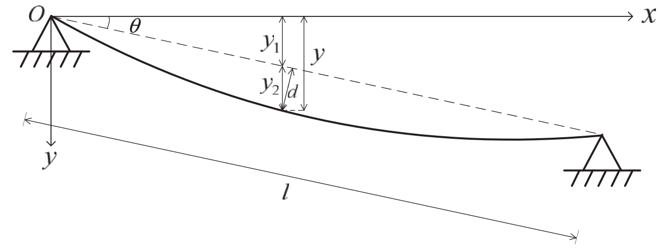

Fig. 2. Inclined cable and its coordinate system.

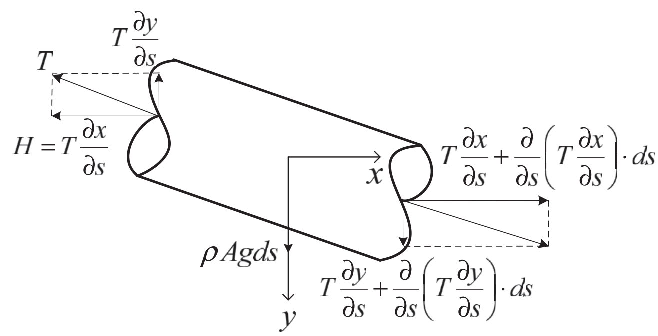

Fig. 3. Differential cable element due to self-weight.

incorporates the strain energy due to tension, bending and torsion (Kim et al., 2010, 2013). The equations of motions of both the mooring cables and the floating platform are solved separately and iteratively. With this approach, the tensile forces applied on the platform by the cables are used to update the equations of motions of the floating platform and then the displacements of the floating platform are used to update the boundary conditions of the mooring cables iteratively until the solution converges at each time step.

In view that the mooring lines behavior more like cables, the full stiffness matrix of the cable element is formulated in this paper. The advantage of using nonlinear cable element rather than nonlinear beam element is that the computational effort can be much reduced. The cable element is formulated based on the extended Hamilton principle (Pai, 2007) and the nonlinear stiffness matrix, rather than the tangent nonlinear stiffness matrix, of the cable is derived and expressed explicitly in order to formulate the equations of motion of the system (Wang et al., 2016). The exact catenary profile of the mooring cables in the static state is determined for a given sag, which is referred to as the cable's initial state. The modeling of the floating platform is simplified as a rigid body with three degrees of freedom, i.e., two translational displacements and one rotational displacement. The hydrodynamic drag forces are taken into consideration and applied to both the mooring cables and the floating platform. The connection conditions between the floating platform and the mooring cables are introduced in deriving the equations of motion for both the floating platform and the mooring cables as a whole. The nonlinear equations of motion of the whole system are then solved by the fourth-order Runge-Kutta method. The derived cable element was validated by comparing the results from the cable element to those from other methods (O'Brien and Francis, 1964; Jayaraman and Knudson, 1981; Yang and Tsay, 2007). The correctness of the solution procedure and the obtained internal forces is validated by examining the equilibrium conditions at the nodes of the system. After that, the displacements of the moored floating structures and the maximum tensile force in cables are investigated for different current velocities, sag-to-span ratios, and inclined angles of the mooring cables. The displacements of the moored floating structure and the maximum tensile force in the cables are also studied with different excitation frequencies and current velocities to identify the critical performances of the structure.

2. Statement of the problem

Consider a two-dimensional moored floating structure as shown in Fig. 1. The structure consists of a floating platform and two catenary mooring cables C_1 and C_2. The upper ends of the mooring cables are connected to the floating platform at two points, A and B, respectively. The lower ends of the mooring cables are fixed on the sea bed. C_1 and C_2 are assumed to be symmetric about the y axis in static state. This model can be used to analyze the vibrations of the moored platform when there is a vertical symmetrical plane and the vibration is in this symmetrical plane.

As shown in Fig. 1, l is the length between the two ends of each mooring cable; θ and d are the inclined angle and the maximum sag of the mooring cables, respectively; D is the center of mass of the floating platform; w_d and h_b are the length and height of the floating platform, respectively; h_s is the submerged height of the floating platform in the sea in static state; h is the depth of the sea. The modeling of the floating platform is simplified as a rigid body with three degrees of freedom, i.e., the displacements at D in the x and y directions and the rotation in the xOy plane about D.

2.1. Catenary profile of mooring cables in static state

The initial profile of the mooring cables in static state is governed by the pretension and self-weight of the cables. The exact catenary profile of the cable is required for a given sag-to-span ratio, d/l, in the following analysis, as shown in Fig. 2.

The following static equilibrium equations in the x and y directions are derived based on the equilibrium of the elements as shown in Fig. 3.

$$\sum F_x = 0 : \frac{\partial}{\partial s}\left(T\frac{\partial x}{\partial s}\right) = 0, \tag{1}$$

$$\sum F_y = 0 : \frac{\partial}{\partial s}\left(T\frac{\partial y}{\partial s}\right) = -\rho Ag, \tag{2}$$

where x and y are the coordinates of a point in the cables in static state due to both the self-weight of the cables and pretension in the cable; T is the tension in the cable in static state; H is the horizontal component of the cable tension in static state; s is the coordinate along the cable length in static state; ρ is the mass density of the cable; A is the cross-section area of the cables.

A location parameter a is introduced which satisfies $dy/dx|_{x=a} = 0$. Solving Eq. (2) with the boundary condition $y|_{x=0} = 0$ and the introduced condition $dy/dx|_{x=a} = 0$ gives the catenary profile of the cables as follows.

$$y = -\frac{H}{\rho Ag}\cosh\left[-\frac{\rho Ag}{H}(x-a)\right] + \frac{H}{\rho Ag}\cosh\left(\frac{\rho Aga}{H}\right). \tag{3}$$

Substituting the boundary condition $y|_{x=l\cos\theta} = l\sin\theta$ into Eq. (3) gives