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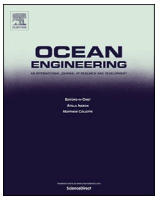

# Dependence of critical filling level on excitation amplitude in a rectangular sloshing tank

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## ABSTRACT

A series of experiments has been conducted to investigate the fluid sloshing at the first mode (i.e. half wavelength exists inside the tank) for different filling levels in a rectangular tank. For each filling level, fluid sloshing was driven by forced tank motions in the horizontal plane with excitation amplitudes of 0.127 cm, 0.254 cm, 0.508 cm and 0.635 cm, respectively. Pressure variations on the side wall of the tank and wave elevation inside the tank were measured simultaneously using four pressure sensors and a wave gauge. It was found that the filling level at which the maximum sloshing amplitude occurs, namely the critical filling level, decreases as the excitation amplitude increases. A theoretical analysis was conducted to provide a physical explanation for this trend. It was also found that an increase in the damping of the system leads to a shift of the critical filling level to a higher value. The pressure on the tank sidewall is found to correlate well with the response amplitude of the water level in the tank since small excitation amplitudes were used in this study.

## 1. Introduction

Sloshing is a flow phenomenon that usually occurs in a partially-filled tank when the excitation frequency approaches one of the natural frequencies of the fluid sloshing modes. When sloshing occurs, the fluid inside the tank is subjected to violent oscillation which may induce significant localized impact pressure on the tank wall. It may further induce structural damages and destabilization to a vessel (Zhao et al., 2014) under extreme conditions. Hence, sloshing phenomenon is of practical interest in offshore oil and gas and shipping industries.

Sloshing has been attracting lots of attentions over the past decades. For rectangular tanks, the natural frequencies of the internal sloshing modes can be calculated through the theoretical relation below (Abramson, 1966),

$$\omega_n^2 = \pi n\left(\frac{g}{L}\right)\tanh\left(\pi n\frac{h}{L}\right), \tag{1}$$

where $n$ is the mode number of the internal sloshing, $L$ is the tank length, $g$ is the gravitational acceleration and $h$ is the filling depth of the tank. Specifically, $\omega_1$ denotes the fundamental frequency (i.e. $n = 1$). Apart from the above theoretical relation, experiments are also often carried out to determine the natural frequencies through free decay tests (Kobayashi et al., 1989). Pal and Bhattacharyya (2010) experimentally investigated the relationship between sloshing amplitude and the excitation frequency. The fundamental sloshing frequencies (the first mode) at different filling levels were found to be different from those calculated by the theoretical method in Eq. (1), with discrepancies of approximately 4%.

Sloshing responses under large and medium excitation amplitudes have been studied extensively in the past decades, for example, the experimental study by Ji et al. (2012) on non-resonant sloshing at a large amplitude (the ratio between the forcing amplitude $b$ and the tank length $L$, $b/L$, was 0.1), and Wei et al. (2015) on slamming pressure inside a sloshing tank with large amplitude excitations ($b/L = 0.102$ to 0.116). Royon-Lebeaud et al. (2007) conducted sloshing experiments in a square tank under a median excitation amplitude of $b/L = 0.0125$. They investigated wave crest destabilization in detail and interpreted the cross-wave instability in terms of parametric instability. Sloshing under small excitation amplitudes has also been studied experimentally. Pal and Bhattacharyya (2010) studied the liquid movement in a sloshing tank under five different excitation amplitudes ($b/L = 0.005$, 0.01, 0.015, 0.02, 0.025, the first one is considered as small while the rest are considered as moderate). Following the sinusoidal variation in the

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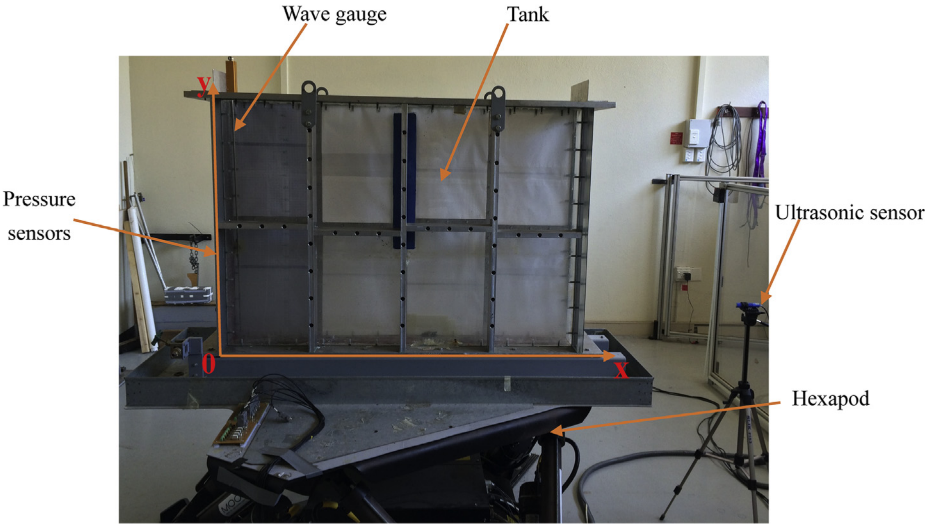

**Fig. 1.** Photo of the sloshing experimental facility and the definition of the co-ordinate system.

external frequency, it was found that the sloshing amplitude also displays a sinusoidal pattern, i.e. the sloshing amplitude increases initially with the increase in applied frequency and decreases thereafter, and again with the increase in external frequency the slosh amplitude increases.

Apart from the excitation amplitude, the filling depth of the liquid inside the tank is another major factor that influences the sloshing response. Low filling depths have attracted much attention. Verhagen and Wijngaarden (1965) observed a hydraulic jump, which travelled periodically back and forth inside the tank with low filling levels under a roll excitation. Antuono et al. (2014) extended the mathematical sloshing model proposed by Antuono et al. (2012) for rectangular tanks under shallow water conditions to study moderate to strong wave breaking from bores to plunger under both sway and roll motions. The kinematics of the flip-through in sloshing in the shallow water regime was studied by Lugni et al. (2006) using Particle Image Velocimetry (PIV). In addition, the features of sloshing for high-filling depth, such as the kinematic and dynamic behaviours after water hits the ceiling of the tank, have also been studied. High filling levels can lead to high impact pressure because of the occurrence of sudden flip-through (Faltinsen and Timokha, 2009). Abrahamsen and Faltinsen (2011) conducted experiments in a tank with high filling levels to study an air pocket entrapped by a free surface wave. Akyıldız et al. (2013) conducted a series of experiments with three different filling levels, namely 25%, 50% and 75%, in a cylindrical tank to elucidate how the filling level influences the response amplitude. They concluded that the liquid sloshing becomes weaker with the increase of the filling level due to the damping effect. Due to the limited number of filling levels studied, it was not possible to elaborate the relationship between the filling level and the sloshing wave amplitude responses.

The majority of the existing studies has focused on the sloshing phenomena at very low or high filling levels with interests in the local nonlinear phenomena. However, there is a practical interest in the relationship between the maximum sloshing motion and filling levels under small excitation amplitudes because this is closely related to safe operations of floating liquefied natural gas (FLNG) production systems in ocean environments. In this study, the critical filling level, $h_{cr}/L$, at which the maximum sloshing response occurs is investigated experimentally, where the rectangular tank is driven horizontally with a given amplitude at the fundamental natural frequency of the internal sloshing over a range of filling levels. The critical filling level for a given excitation amplitude is then quantified and the physical explanation responsible for this phenomenon is explored.

## 2. Experimental set-up

The experiments were conducted in the Sloshing Laboratory at The University of Western Australia. Fig. 1 shows the experimental set up and the definition of the co-ordinate system, including the 6DOF hexapod, the 2D tank, the ultrasound sensor and the wave gauge used to detect the

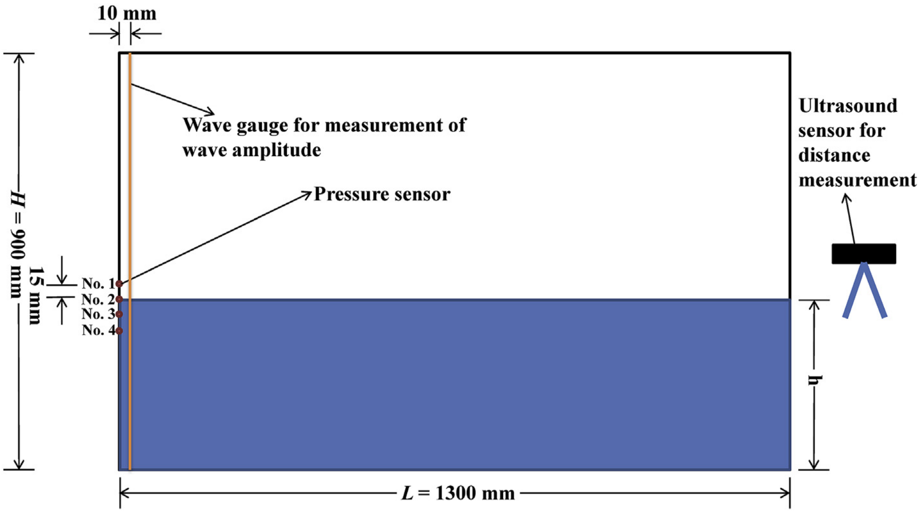

**Fig. 2.** Sketch of the tank and the arrangements of the sensors.