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# Prediction of droplet size and velocity distribution for spray formation due to wave-body interactions

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ABSTRACT

This study addresses the development of a statistical tool for predicting the distribution of the size and velocity of droplets in spray caused by the interaction of waves with a marine object. The Maximum Entropy Principle (MEP) is a statistical tool that allows the prediction of a probability distribution that is consistent with information from the input system. The prediction satisfies constraint equations covering the conservation of mass, momentum, and energy. The velocity distribution of droplets is Gaussian in shape. The effect of a drag force on both the liquid sheet that is formed from the wave impact as well as the downstream distribution of droplets was considered in this simulation. The examination of the mechanisms of turbulence diffusion in a wave at the moment of impact with an object provides a logical, analytical relationship between the wave flows and a spray cloud formation after impact. The model prediction is compared with the experimental data of spray-cloud formation due to wave impact from a lab-scale model, and is found to be in good agreement. The prediction model is then applied to the full-scale model based on the data from previous field observations to predict the droplet size and velocity distributions of spray cloud due to the wave interactions with the vessel.

## 1. Introduction

Wave spray events are recognized as a dangerous occurrence for marine vessels and offshore structures. Spray events are characterized by dispersed masses of liquid that move to the vessel deck after a wave impact. Wave spray combined with cold air temperature cause significant icing conditions on these structures, which can affect stability, structural integrity, and increase safety risks and operation hazards on board (Roebber and Mitten, 1987; Ryerson, 2013; Bodaghkhani et al., 2016; Dehghani-Sanij et al., 2015). The simulation of wave spray is a complex procedure, which consists of simulating several free-surface related events, such as breaking waves, water sheet breakup, and spray formation. These phenomena are currently among the most challenging problems in computational fluid dynamics (Hendrickson et al., 2003; Dehghani-Sanij et al., 2018).

In the framework of large ships and structures, the amount of water delivery, droplet size, velocity distributions, and height of the spray are major concerns (Bodaghkhani et al., 2017). Although wave spray is the major cause of icing on vessels, the physical nature of wave impact and spray formation are still poorly understood. Besides a few observational studies, which provide a qualitative understanding of the water delivery and spray heights, little research on this subject is available in the literature (Dehghani-Sanij et al., 2017a, 2017b).

Modeling the distribution of droplet size and velocity is essential in the study of spray cloud formation, and these distributions are crucial parameters for the fundamental analysis of droplet trajectories upside of the bow of a vessel (Dehghani et al., 2016a, Dehghani and Naterer, 2016b). Typical analytical models for predicting the size and velocity of droplets are extracted from experimental data and measurements for small-scale events, such as spray formation from a nozzle. Examples of these distributions are the Rosin-Rammler distribution (Rosin and Rammler, 1933), and the Nukiyama-Tanasawa distribution (Nukiyama and Tanasawa, 1939). The interested reader can refer to a review paper by Babinski and Sojka (2002) for broad spray distribution models.

More recent studies use statistical approaches to predict more general droplet sizes and velocity distributions. The MEP method has become popular for the prediction of droplet sizes and velocity distributions because it produces reasonably accurate results. Sellens and Brzustowski (1985, 1986), and Li and Tankin (1987, 1988) were the first to introduce this method. Since the theory of this method has been well described by

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these researchers, the background materials are not covered in this article. The method assumes that while the system entropy is maximized and conservation equations (mass, momentum, and energy) are satisfied, the equation for size distribution will be equally satisfied. The MEP method has been upgraded for different conditions, and the effects of several phenomena were added over the past two decades. Kim et al. (2003) used the MEP and instability analysis of liquid sheets to consider the effect of the instability of liquid jets in their model. Huh et al. (1998) consider the effect of turbulent conditions for diesel sprays in conjunction with the MEP model.

The MEP was originally introduced for predicting the droplet size and velocity distribution of diesel spray from a nozzle. However, in this study, the authors apply the MEP to model the spray production due to wave interaction with marine objects. Similarly, after the wave impact, the water forms a liquid sheet, which is the same as the non-homogenous, inverted hollow cone that exits from a nozzle with one side of the cone attached to the marine structure. Afterward, it breaks up into ligaments that form droplets. This new model prediction was compared with the results of droplet and velocity measurements from wave impacts on a lab-scaled, flat-shaped and bow-shaped plate models. In this experiment, the Bubble Image Velocimetry (BIV) method was used to measure droplet size and velocity across a selected plane in the spray. Moreover, these results are compared with full-scale data from the field observations of Ryerson (1995).

The experiment was designed for measuring droplet size and velocity distribution across a plane in front of two models: the flat-shaped and the bow-shaped plate models. The BIV technique was used to illuminate the spray for further post-processing measurements. This technique was first introduced by Ryu et al. (2005) to experimentally analyze the kinematics of plunging breaking waves that impinge and overtop a structure; it is based on the principles of Particle Image Velocimetry (PIV). The BIV technique was used to measure the velocity fields in the aerated region around the structure. A study using BIV techniques to measure the flow kinematics inside an aerated area can be found in Govender et al. (2002). Other attempts to measure flow kinematics outside of aerated areas using the BIV method are reported by Chang and Liu (1999, 2000) and Melville et al. (2002). Chang and Liu (1998) used PIV to measure the velocity of the overturning jet of breaking waves. The flow characteristics of aerated regions are rarely reported by researchers. Some applications of the BIV method to model green water and sloshing can be found in Ryu et al. (2007a, 2007b), Chang et al. (2011), and Song et al. (2013).

In this paper, new mathematical formulations applied to the MEP method are described. Two drag coefficient equations are introduced based on the shape of the water sheet after the wave impact as well as drag coefficient for the droplets. In the next section, experimental models and methods are described. Then, the results of the predicted model are compared with the results of the experimental data. Lastly, the MEP prediction model is applied to a full-scale scenario.

## 2. Mathematical formulations

Spray formation from wave impact is the result of a thin sheet of liquid, which develops instabilities and breaks up into ligaments, and lastly, forms droplets. The thermodynamic laws for when equilibrium states are transferred from one state to another govern the size distribution and velocity distribution of droplets. During this transformation, the equations for mass, momentum, and energy conservation are used as constraints, while entropy maximization occurs. Constraints are developed by assuming that the breakup and spray formation is a practically conservative process. The conservation equations downstream of a wave impact area can be presented as the probability density function $f$, which is the probability of finding droplets based on both droplet diameter $D$ or droplet volume $V_d$, and droplet velocity $v_d$. In this method, it is assumed that the droplets formed just downstream of the breakup area have the same total mass, momentum, kinetic energy, and surface energy as the primary water sheet.

The solution for all constraints contains both of these variables so that $d\varphi = d\overline{v}d\overline{D}$. The solution domain $d\varphi$ consist of both $\overline{v}$, velocity characteristics, and $\overline{D}$, diameter characteristics. According to the probability concept, the total summation of probabilities is equal to unity $\sum_i\sum_j f_{ij} = 1$. By combining all the conservation constraints with the normalization constraints, information regarding droplet size and velocity distributions based on the conservation laws can be provided (Sellens and Brzustowski, 1985). The following are a normalized set of equations (Li and Tankin, 1987; Li et al., 1991) that can be solved iteratively based on the Newton-Raphson procedure to predict a size and velocity distribution model for spray cloud.

$$\int_{\overline{D}_{min}}^{\overline{D}_{max}}\int_{\overline{v}_{min}}^{\overline{v}_{max}} f\overline{D}^3 d\overline{v}d\overline{D} = 1 + \overline{S}_m \tag{1}$$

$$\int_{\overline{D}_{min}}^{\overline{D}_{max}}\int_{\overline{v}_{min}}^{\overline{v}_{max}} f\overline{D}^3 \overline{v}d\overline{v}d\overline{D} = 1 + \overline{S}_{mu} \tag{2}$$

$$\int_{\overline{D}_{min}}^{\overline{D}_{max}}\int_{\overline{v}_{min}}^{\overline{v}_{max}} f\left(\overline{D}^3\overline{v}^2\Big/H + B\overline{D}^2\Big/H\right) d\overline{v}d\overline{D} = 1 + \overline{S}_e \tag{3}$$

$$\int_{\overline{D}_{min}}^{\overline{D}_{max}}\int_{\overline{v}_{min}}^{\overline{v}_{max}} f d\overline{v}d\overline{D} = 1 \tag{4}$$

$$f = 3\overline{D}^2 \exp\left[-\lambda_0 - \lambda_1\overline{D}^3 - \lambda_2\overline{D}^3\overline{v} - \lambda_3\left(\overline{D}^3\overline{v}^2\Big/H + B\overline{D}^2\Big/H\right)\right] \tag{5}$$

Equations (1)–(3) are mass conservation, momentum, and energy equations, respectively. Eq. (4) is a normalization constraint, and Eq. (5) is the Probability Density Function (PDF). In Eq. (3), the $\overline{S}_e$ represents all the sources of energy. However, energy conversion was not considered in this simulation. In Eq. (5) $B = 12/We$ $We = \rho U_0^2 D_{30}/\sigma$, and $H$ is the shape factor, which is equal to 1 for the uniform velocity profile.

In these equations, the solution domain changes from $\overline{D}_{min}$ to $\overline{D}_{max}$ for droplet size variations, and from $\overline{v}_{min}$ to $\overline{v}_{max}$ for droplet velocity variations. These variables are set as 0 to 1500 $\mu m$ for droplet sizes and 0 to 8 $m/s$ for droplet velocities. According to Li and Tankin (1987), the dimensionless terms in this set of equations were introduced as $\overline{D}_i = D_i/D_{30}$, $\overline{v}_j = v_j/U_0$, where $U_0$ is the average initial velocity of the water sheet at the moment of impact with the wall, and $D_{30}$ is the mass mean diameter of droplets and was calculated based on the experimental results using the following equation:

$$D_{30} = \sum m_i d_i\Big/M = \sum n_i d_i^4\Big/\sum n_i d_i^3 \tag{6}$$

In Eq. (5), $f$ is the PDF for representing the continuous size and velocity distribution in an integral form. In Eq. (1), the non-dimensional mass source term ($\overline{S}_m$) was neglected because the effect of evaporation and condensation was not considered. The drag force was considered to be acting on the liquid sheet and spray droplets. The drag force on the liquid is calculated as a momentum transformation, and is considered as a momentum source. By considering that the drag force is acting on a single side of the liquid sheet with a length of $L_b$ (the other side is attached to the model), the drag force ($F_1$) is written as:

$$F_1 = {}^1\!/\!_2\rho_a U_0^2 A_f C_D \tag{7}$$

where $C_D$ is the drag coefficient of flow over a flat plate with a contact area of $A_f$ (White, 1991) and is calculated based on the following equation:

$$\begin{cases} C_D = 1.328/\sqrt{Re_L} & 10^3 < Re_L \\ C_D = \left(1.328/\sqrt{Re_L}\right) + 2.3/Re_L & 1 < Re_L < 10^3 \end{cases} \tag{8}$$