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Boundary control of transverse motion of flexible marine risers under stochastic loads

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ABSTRACT

A constructive design of boundary controllers is proposed to stabilize transverse motion of flexible marine risers under stochastic loads induced by restoring membrane and fluid/air velocity. For stability analysis and boundary control design, global well-posedness (existence and uniqueness) and global stability criteria are developed for nonlinear stochastic evolution systems in Hilbert space subject to both state-dependent and additive stochastic disturbances. These criteria are obtained by developing Lyapunov sufficient conditions from local well-posedness and stability results. The well-posedness and stability developments are not only applied to ensure well-posedness and almost sure (practical) stability of the variational (strong) solution of the stochastic riser system but also to other hyperbolic systems.

1. Introduction

As offshore exploration and production of natural resources such as oil and gas enters ever-deeper waters, production risers transporting petroleum products from the sea-bed to a surface ship/rig become extremely slender (the ratio of average diameter to length can be of the order 10^{-5}), and are subject to larger nonlinear vibration and deformation due to stochastic environmental loads, see Faltinsen (1993). Excessive vibration of the risers can cause premature fatigue and loops, and shut down of the system, see Sorensen (2005), which result in both devastating environmental and financial consequences.

A representative riser system is given in Fig. 1, where the riser is connected to the well at seabed and ship/rig via ball joints. Reducing vibration of risers can be carried out by active control systems and/or passive devices. Passive systems are well established and limited to shallow water due to drag, see Baek and Karniadakis (2009). This paper focuses on active control systems installed at the riser top-end. As such, this top-end is connected to an active heave compensation system, see Do and Pan (2008b). The horizontal motion is controlled by a dynamic positioning system actuated by the ship/rig thrusters, see Sorensen (2005), or an actuated guide tube mechanism.

In existing works, the riser is modeled by a distributed parameter system (DPS) governed by hyperbolic partial differential equations (PDEs). There are two typical approaches to the control design. In the modal control approach, a lumped parameter system (LPS) governed by a set of ordinary differential equations (ODEs), described in terms of modal coordinates, is first obtained from the DPS by the Galerkin or assumed modes method. Then, classical control design methods, see Anderson and Moore (1990); Krstic et al. (1995); Khasminskii (1980); Deng et al. (2001); Mao (2007); Do (2015), can be applied to the LPS. This approach can only control a certain number of modes of a DPS and suffers from a spill-over problem, see Meirovitch (1997).

In the boundary control approach, the DPS is directly considered and controllers are implemented at the boundaries to control all the modes. Using the Lyapunov direct method in Khalil (2002), various boundary controllers of a proportional-derivative type have been proposed for flexible beam-like systems such as pipes and risers, see Stanway and Burrows (1981); Fard and Sagatun (2001); Fung et al. (1999); Fung and Tseng (1999); Do and Pan (2008a, 2009); Do (2011); Ge et al. (2010); He et al. (2011); Nguyen et al. (2013); Bohm et al. (2014); Do (2017, 2016). There is serious problem with all of the above works: the environmental loads on the riser were assumed to be deterministic. However, the loads induced by ocean currents, waves and wind are stochastic in nature Faltinsen (1993). This results in motion of the riser being described by a stochastic distributed-parameter system (SDPS) governed by stochastic partial differential equations (SPDEs). Well-posedness (existence and uniqueness) and stability of stochastic beams under and Lipschitz conditions subject to Dirichlet/Neumann boundary conditions (without boundary control) have been addressed by several mathematician, see Zhang (2007); Brzezniak et al. (2005); Chow and Menaldi (2014); Chow (2007). Various well-posedness and stability results developed for stochastic parabolic PDEs, see Pardoux (1979); Liu (2006); Prato and

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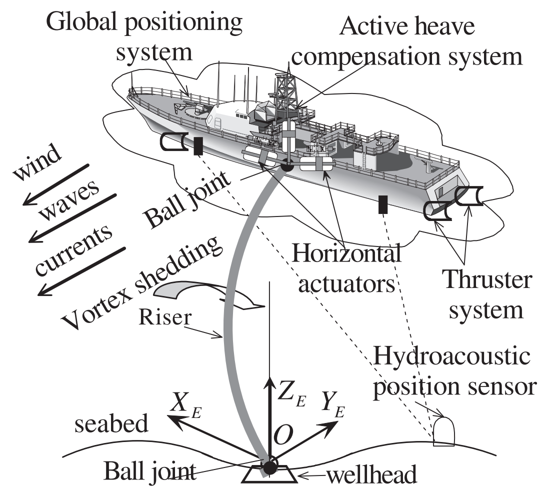

Fig. 1. A representative riser system.

Zabczyk (1992); Liu and Mandrekar (1997); Gawarecki and Mandrekar (2011) and references therein are not directly applicable to analyze motion of the marine risers (flexible beams) under stochastic loads since motion of the risers are described by stochastic hyperbolic PDEs. This issue is mainly due to difficulty when checking coercivity and growth conditions.

The above discussion motivates the writing of this paper on a design of boundary controllers stabilizing transverse motion of stochastic flexible marine risers. The control design and stability analysis are based on Lyapunov direct method, which is developed in this paper, for stochastic evolution systems (SESs) in Hilbert space to guarantee almost sure (a.s.) global practical stability of the variational (strong in the stochastic sense but weak in the PDE) solution of the marine riser transverse motion system. Since, in addition to marine risers, various systems encountered in practice, such as elastic plates, panels, shells, strings, and membranes, see Dowell (1975), are hyperbolic systems subject to both state-dependent and additive stochastic disturbances. Global well-posedness (existence and uniqueness) and a.s. global stability criteria are developed for nonlinear SESs subject to both state-dependent and additive stochastic disturbances.

2. Problem formulation

2.1. Transverse motion of stochastic risers

Assume that the plane sections of the beam remain plane after deformation (i.e., warping is neglected); the riser is locally stiff (i.e., cross sections do not deform and Poisson effect is neglected); the beam material is homogeneous, isotropic and linearly elastic (i.e., it obeys Hookes's law); torsional and distributed moments induced by environmental disturbances are neglected; and that the beam deforms in one vertical plane, and its axial motion is ignored. Based on the Hamiltonian principle, the equations of motion of the riser with a configuration as shown in Fig. 2 where the earth-fixed coordinate system is OYZ and both of the riser ends are connected to the fixed base and the ship/rig via ball joints, see Do and Pan (2008a), can be derived as

$$\begin{aligned} \varrho a u_{tt}(z,t) &= -EIu_{zzzz}(z,t) + \left(P_0 + \frac{3Ea}{2}u_z^2(z,t)\right)u_{zz}(z,t) \\ &\quad + q(z,t,u(z,t)-\vartheta(z,t), u_t(z,t)-\nu(z,t)), \\ &\quad -EIu_{zzz}(L,t) + P_0 u_z(L,t) + \frac{Ea}{2}u_z^3(L,t) = \phi_B, \\ &\quad u_{zz}(0,t)=0, u_{zz}(L,t)=0, u(0,t)=0, \\ &\quad u(z,t_0)=u_0(z), u_t(z,t_0)=u_1(z), \end{aligned} \tag{1}$$

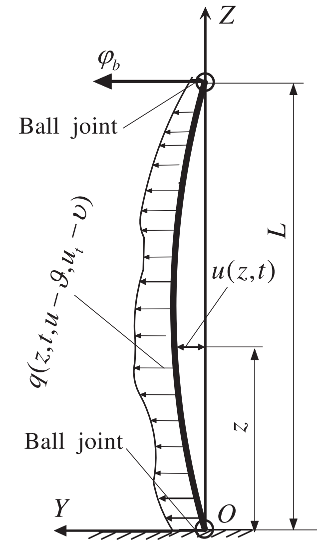

Fig. 2. Riser coordinates.

where ϱ is the mass density; a is the cross section area of the riser; $u(z,t)$ denotes the transverse displacement of the riser at (z,t); $u_t(z,t)$ and $u_z(z,t)$ denote the partial derivatives of $u(z,t)$ with respect to time t and the spatial coordinate z, and similarly for notations $u_{tt}(z,t)$, $u_{zz}(z,t)$, and $u_{zzzz}(z,t)$; E is Young's modulus; I is the moment of inertia of the riser cross section; P_0 is the constant axial force; $u_0(z)$ and $u_1(z)$ are initial values; ϕ_B is the boundary control force; and $q(\bullet)$ with $\bullet$ being for $(z,t, u(z,t)-\vartheta(z,t), u_t(z,t)-\nu(z,t))$ is the distributed force. This distributed force (nonlinearly) depends on the relative displacement $u(z,t)-\vartheta(z,t)$ between the riser and the membrane/spring (Elosta et al. (2013)), which is connected to the riser, and relative velocity $u_t(z,t)-\nu(z,t)$ between the riser and the fluid/air particle. In this paper, the distributed force $q(\bullet)$ is modelled by

$$\begin{aligned} q(\bullet) &= -c_1(z,t)(u(z,t)-\vartheta(z,t)) - c_2(z,t)(u(z,t)-\vartheta(z,t))^3 \\ &\quad -d_1(z,t)(u_t(z,t)-\nu(z,t)) - d_2(z,t)(u_t(z,t)-\nu(z,t))^3, \end{aligned} \tag{2}$$

where $c_1(z,t)$ and $c_2(z,t)$ are referred to as the restoring coefficients while $d_1(z,t)$ and $d_2(z,t)$ are referred to as the damping coefficients. It is noted that the distributed force usually includes non-smooth restoring and damping terms such as $-c_0(z,t)|u(z,t)-\vartheta(z,t)|(u(z,t)-\vartheta(z,t))$ and $-d_0(z,t)|u_t(z,t)-\nu(z,t)|(u_t(z,t)-\nu(z,t))$ with $c_0(z,t)$ and $d_0(z,t)$ being restoring and damping coefficients. However, these terms can be dominated by those terms already included in (2). Thus, we do not include the non-smooth terms in the distributed force $q(\bullet)$.

In general, the membrane displacement and air/fluid velocity consist of deterministic and stochastic parts, i.e., $\vartheta(z,t)$ and $\nu(z,t)$ can be written as

$$\begin{aligned} \vartheta(z,t) &= \vartheta_d(z,t) + \vartheta_s(z,t), \\ \nu(z,t) &= \nu_d(z,t) + \nu_s(z,t), \end{aligned} \tag{3}$$

where $\vartheta_d(z,t)$ and $\vartheta_s(z,t)$ are referred to as the deterministic and stochastic parts of $\vartheta(z,t)$, respectively; and similar notations for $\nu_d(z,t)$ and $\nu_s(z,t)$. Substituting (3) into (2) gives

$$q(\bullet) = f(z,t,u(z,t),u_t(z,t)) + q_s(z,t,u(z,t),u_t(z,t),\vartheta_s(z,t),\nu_s(z,t)). \tag{4}$$

The deterministic part $f(\bullet)$ of $q(\bullet)$ is given by

$$\begin{aligned} f(\bullet) &= -a_1(z,t)u(z,t) + a_2(z,t)u^2(z,t) - a_3(z,t)u^3(z,t) \\ &\quad -b_1(z,t)u_t(z,t) + b_2(z,t)u_t^2(z,t) - b_3(z,t)u_t^3(z,t) + f_0(z,t), \end{aligned} \tag{5}$$

where