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A multiwavelet support vector regression method for efficient reliability assessment

Hongze Dai [a,b,*], Boyi Zhang [a,b], Wei Wang [a,b]

[a] School of Civil Engineering, Harbin Institute of Technology, Harbin 150090, China
[b] Key Lab of Structures Dynamic Behavior and Control (Harbin Institute of Technology), Ministry of Education, Harbin 150090, China

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ABSTRACT

As a new sparse kernel modeling technique, support vector regression has become a promising method in structural reliability analysis. However, in the standard quadratic programming support vector regression, its implementation is computationally expensive and sufficient model sparsity cannot be guaranteed. In order to mitigate these difficulties, this paper presents a new multiwavelet linear programming support vector regression method for reliability analysis. The method develops a novel multiwavelet kernel by constructing the autocorrelation function of multiwavelets and employs this kernel in context of linear programming support vector regression for approximating the limit states of structures. Three examples involving one finite element-based problem illustrate the effectiveness of the proposed method, which indicate that the new method is efficient than the classical support vector regression method for response surface function approximation.

1. Introduction

Structural reliability analysis seeks to obtain the probability of an event typically related to a possible failure of various engineering systems. For most structures of practical interest, the limit state cannot be expressed as explicit, closed-form function of the input random variables because the structural responses have to be determined by a numerical procedure such as finite element (FE) analysis. This brings great challenges for assessing the failure probability of realistic structural systems. A common technique for analyzing structural reliabilities with complex/implicit limit state functions is to use the surrogate model method. It uses a strategic design of experiments to obtain an analytical approximation of the relationships between the input random variables and the limit state response of interest. The earlier application of this approach is the use of the response surface methods [1,2]. However, the results given by the response surface methods may be sensitive to the sample selection, the interpolation polynomial, or the shape of the limit state due to the rigid and non-adaptive structure of the polynomial models [3,4].

In order to overcome the above limitations, artificial neural networks (ANNs) have been proposed as an alternative tool for estimating response surface functions. The beauty of ANNs is their flexibility in nature and their ability to capture complex nonlinear relationships between input and output through appropriate learning. It has been shown that the ANNs' models have practical advantages over the classical response surface method because of their superior mapping capacities and the flexibility in functional form [5–8]. However, the performance of ANNs cannot be guaranteed due to the fitting problems because there is no efficient constructive method for choosing the structure and the learning parameters of the network [9]. The main challenge for ANNs model is to suitably choose the learning parameters that help restrain under- or over-fitting, as both are equally disastrous [10,11].

As a novel sparse kernel modeling technique, support vector machine (SVM) has been gaining popularity in the field of machine learning during the past decade [12]. The standard SVM is optimized by solving a linearly constrained quadratic programming (QP) problem so that the solution of SVM is globally optimal and unique, and the non-linear ability of SVM is achieved by using the kernel mapping. When SVM is employed to deal with the function approximation problems, it is often referred to as the support vector regression (SVR). Compared with ANNs, SVM uses the theory of minimizing the structure risk to avoid the problems of excessive study, calamity data, local minimum value and so on. Even though SVM has exhibited excellent performance, the literature on the use of SVM for structural reliability analysis is still limited. Hurtado [13,14] employed SVM as a classifier to discriminate samples into

* Corresponding author at: School of Civil Engineering, Harbin Institute of Technology, Harbin 150090, China. Tel.: +86 451 86282075.
E-mail address: hzdai@hit.edu.cn (H. Dai).

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safe and failure classes in structural reliability analysis. SVM was also employed as a regression tool for approximating the original limit state for reliability analysis [15–18]. In addition, Dai et al. [19] proposed a support vector density-based importance sampling method for structural reliability analysis more recently, in which SVM was employed as a density estimator to construct the importance sampling density.

As a summary most of the current practice employs the standard SVM as statistical classifier or function approximator in structural reliability analysis. However, in the standard QP-SVM, the regression function obtained often contains redundant terms and the inefficiency of QP-SVM for selecting support vectors could lead to infeasible models. This is particularly apparent in regression application where the entire training set can be selected as support vectors if error insensitivity is not included [20]. On the other hand, since the commonly used kernel functions, such as Gaussian kernel or polynomial kernel, is not the complete orthonormal bases, the SVM cannot approach any curve in quadratic continuous integral space $L^2(\mathbb{R})$ [21]. Therefore, a good choice of the kernel plays a critical role in the performance of SVM.

In order to mitigate the above difficulties, various improvements of support vector algorithms and kernels were developed in recent years. For example, Smola and Schölkopf proposed a linear programming (LP) SVM to control the accuracy and sparseness of QP-SVM by using the linear kernel combination as an ansatz for the solution, and employing l_1 norm of the coefficient vector in the cost function [22]. Suykens and Vandewalle proposed a least square (LS) SVM to improve the performance of standard SVM by transforming the QP to a linear equation sets [23]. Zhang et al. introduced the wavelet kernel into QP-SVM and found that it outperforms the Gaussian kernel in function regression since wavelet function is orthonormal in $L^2(\mathbb{R})$ [24]. Although the above improved SVM has been successfully applied in many fields, their application to structural reliability analysis is rather limited. Guo and Bai employed the LS-SVM to approximate the limit state in structural reliability analysis [25]. Tan et al. compared the ANN-based response surface and LS-SVM-based response surface method for structural reliability analysis [26]. Khatibinia et al. proposed a wavelet weighted LS-SVM model to approximate the limit state in seismic reliability analysis of structure, in which the Morlet wavelet function was used as the kernel of the weighted LS-SVM [27].

In this paper, for the purpose of developing an innovative and efficient support vector regression model for limit state function approximation, the issue of model sparsity of standard QP-SVR is addressed from two different perspectives. Firstly, LP-SVR is employed to capitalize on the advantages of the model sparsity, the flexibility in using more general kernel functions, and the computational efficiency of LP, as compared to QP-SVR. Secondly, an innovative multiwavelet kernel is developed by constructing the autocorrelation function of the multiwavelets and this new kernel is then employed in the context of LP-SVR to yield a more compact and sparse representation by leveraging the flexibility of LP-SVR in choosing the kernels. It is known that multiwavelets have many properties that the scalar wavelets do not have. It can simultaneously possess several desirable properties such as orthogonality, regularity and symmetry, while a scalar wavelet cannot possess all these properties at the same time [28,29]. Therefore, it might be expected that the LP-SVR with multiwavelet kernel has a better performance. In addition, special attention is paid to the exploration of the proposed regression model in approximating the complex limit state for structural reliability analysis. To the best knowledge of the authors, the wavelet kernels used in most of the literatures are the scalar wavelet functions, and wavelet kernels are commonly used in context of the standard QP-SVR. The development of multiwavelet kernel and the application of this new kernel in context of LP-SVR have not been studied. This work made a new contribution on this regard.

The paper is organized as follows. Multiwavelet analysis is briefly introduced in Section 2, followed by the description of the proposed multiwavelet LP-SVR model in Section 3. The procedure of the proposed method is then summarized in Section 4. Three examples are then given to demonstrate the application and efficiency of the proposed method. Comparisons of the proposed method, the standard QP-SVM with Gaussian and scalar wavelet kernel, and the LS-SVM with Gauss kernel are made.

2. From wavelets to multiwavelets

Wavelets, also called scalar wavelets, are oscillatory, compactly supported functions that are constructed to possess certain properties such as orthogonality, smoothness and symmetry. Wavelet theory can be introduced using the multiresolution analysis (MRA) developed in [30]. In the framework of scalar wavelets, an MRA is generated by one scaling function $\phi(t)$, and an orthonormal basis for $L^2(\mathbb{R})$ is formed via translation and dilation of one (mother) wavelet $\psi(t) \in L^2(\mathbb{R})$ [30,31].

As an extension of scalar wavelets, multiwavelets consist of several wavelet functions $\underline{\psi}(t) = \{\psi_1(t), \ldots, \psi_r(t)\}$, which are generated from the multiscaling functions $\underline{\phi}(t) = \{\phi_1(t), \ldots, \phi_r(t)\}$, in which r is called multiplicity. A multiwavelet basis for $L^2(\mathbb{R})$ is composed of the scaled translates and dilates of multiwavelet functions $\underline{\psi}(t)$. Because multiwavelets employ multiple scaling functions and multiple mother wavelets, there is more freedom to design these functions to satisfy a greater range of properties, including orthogonality, symmetry, compact support and vanishing moments [28,29]. These properties are very desirable in many applications but cannot be achieved by a scalar wavelet simultaneously.

The multiwavelet dilation equation has the same form as the wavelet dilation equation, except the recursion coefficients $C[k]$ (the multifilter) are $r \times r$ matrices instead of scalars:

$$\underline{\phi}(t) = \sum_{k \in \mathbb{Z}} C[k]\underline{\phi}(2t-k) \quad (1)$$

Like in the scalar wavelet case, the recursion coefficients $C[k]$ can be designed such that the resulting multisclaing function $\underline{\phi}(t)$ is orthogonal across its integer translates. The elements in $C[k]$ provide more degrees of freedom than a traditional scalar wavelet. These extra degrees of freedom can be used to incorporate useful properties into the multiwavelets, such as orthogonality, symmetry, and vanishing moments, which are known to be important for function approximation. Thus, multiwavelets can simultaneously provide perfect reconstruction while preserving length (orthogonality), good performance at the boundaries (via linear-phase symmetry), and a high order of approximation (vanishing moments).

The multiwavelet functions $\underline{\psi}(t)$ are related to the multiscaling function $\underline{\psi}(t)$ via an equation of the same form as in the scalar wavelet:

$$\underline{\psi}(t) = \sum_{k \in \mathbb{Z}} D[k]\underline{\phi}(2t-k) \quad (2)$$

the coefficients $D[k] \in \mathbb{R}^{r \times r}$, however, are not related to the multifilter $C[k]$ by any elegant formula as in the scalar wavelet case. Nevertheless, the dilation equations (1) and (2) state that the multiwavelets and the multiscaling functions are formed as linear combinations of the scaling functions with half of the support.

A number of techniques for constructing multiwavelet have been studied, while theoretical studies on multiwavelets with multiplicity $r > 2$ are rare. In this study, for the purpose of simplicity, we will be concerned only with multiwavelet system with $r=2$, which was constructed by Geronimo, Hardin and Massopust (GHM) [32]. The