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Capacity loss and residual capacity in weighted k-out-of-n:G systems

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ABSTRACT

A binary weighted-k-out-of-n:G system is a system that consists of n binary components, and functions if and only if the total weight of working components is at least k. The performance of such a system is characterized by its total weight/capacity. Therefore, the evaluation of the capacity of the system is of special importance for understanding the behavior of the system over time. This paper is concerned with capacity loss and residual capacity in binary weighted-k-out-of-n:G systems. These measures are potentially useful for the purposes of preventive action. In particular, recursive and non-recursive equations are obtained for the mean capacity loss and mean residual capacity of the binary weighted-k-out-of-n:G system while it is working at a specific time. The mean residual capacity after the failure of the system is also studied.

1. Introduction

Systems with weighted components are applicable to many capacity based engineering problems. There are many situations such that system's components contribute differently to the capacity of the system. Consider a system consisting of n binary components, each with its own positive weight. The weight of a binary component might be assumed to be its performance rate when it is in a functioning state. Assume that the system works if and only if the total weight of working components is above a given threshold k. Such a system is known to be weighted-k-out-of-n:G system. This system model is interesting and useful since it considers the individual impact of its components on the functioning of the system. Consider an oil transportation system consisting of three pipes (components) for transporting oil from one point to another point. The pipes' performance is measured by their transmission capacity (tons per minute). Assume that the pipes can be in two states at any time as either fully operational or complete failure. Such a system can be modeled by weighted-k-out-of-3:G system, where k represents the minimum transmission demand and the weights of the components correspond to the transmission capacities of the pipes. A power generation system that consists of generators with different capacities can also be modeled by a weighted-k-out-of-n:G system. In this case, the weight of a generating unit corresponds to the generation capacity of the unit, and k is the required demand [1].

The above-mentioned systems are also known to be threshold systems in the literature. Reliability modeling and analysis of threshold systems have been considered in various papers including Rushdi [1], Ball et al. [2], and Xie and Pham [3]. The threshold systems which allow more than two performance levels for its components are well presented in Levitin [4].

Wu and Chen [5] (see also [6]) provided recursive equations to compute the reliability of weighted-k-out-of-n:G systems. Chen and Yang [7] introduced and studied two-stage weighted k-out-of-n systems. Li and Zuo [8] compared recursive and universal generating function based methods for reliability evaluation of binary weighted-k-out-of-n:G systems. Samaniego and Shaked [9] presented a detailed analysis on lifetime based analysis of systems with weighted components. Eryilmaz [10] studied residual performance of the systems with weighted components after successive component failures. Armutkar and Kamalja [11], Kamalja and Armutkar [12], and Eryilmaz and Bozbulut [13] studied reliability and importance measures of weighted-k-out-of-n:G systems. Estimation problem for this kind of system has been considered in Aboalkhair et al. [14]. Multi-state extensions of weighted-k-out-of-n:G systems have been considered in Li and Zuo [15], Ding et al. [16], Wang et al. [17], Levitin [18], Eryilmaz and Bozbulut [19], and Faghih-Roohi et al. [20].

Let $T_1, \ldots, T_n$ denote the lifetimes of the components. If the weight of the ith component is denoted by ω_i, then the total weight of the system at time t can be described by the stochastic process:

$$W_n(t) = \sum_{i=1}^{n} \omega_i I(T_i > t), \tag{1}$$

where $I(T_i > t) = 1$ if $T_i > t$ and 0, otherwise. Clearly, the failure time of the system is

$$T = \inf\{t : W_n(t) < k\}.$$

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Because the performance of the weighted-k-out-of-n:G system is characterized by its total capacity, the dynamic reliability measures based on the total capacity $W_n(t)$ of the system are useful in order to understand the behavior of the system over time. Indeed, the reliability of the system at time t is the probability that the total weight of the system at time t is at least k, i.e.

$$R_{n,k}(t) = P\{T > t\} = P\{W_n(t) \geq k\}.$$

If the components are independent, and $P\{T_i \leq t\} = F_i(t), i = 1, \ldots, n$, then the survival function $R_{n,k}(t)$ can be computed from the recursion:

$$R_{n,k}(t) = R_{n-1,k-\omega_n}(t)\overline{F}_n(t) + R_{n-1,k}(t)F_n(t), \tag{2}$$

for $n \geq 1, 0 < k \leq \sum_{i=1}^{n} \omega_i$, with $R_{n,k}(t) = 1$ for $k \leq 0$ [5].

Eryilmaz and Sarikaya [21] obtained the following nonrecursive equation for the particular case when the system consists of two types of elements such that m components have the common weight ω and survival function $\overline{F}(t)$, and the remaining $n-m$ components have the common weight ω^* and survival function $\overline{G}(t)$:

$$P\{T > t\} = \sum_{\substack{\omega y + \omega^* z \geq k \\ 0 \leq y \leq m, 0 \leq z \leq n-m}} \sum \binom{m}{y} \overline{F}^y(t) F^{m-y}(t) \binom{n-m}{z} \overline{G}^z(t) G^{n-m-z}(t), \tag{3}$$

for $t > 0$. Obviously, Eq. (3) is computationally more efficient than (2) if there are only two types of components in the system:

Define the following conditional random variables:

$$(W_n(s) | T > s), \tag{4}$$

and

$$(W_n(t) - W_n(s) | T > s), \tag{5}$$

for $t < s$.

The random variable defined by (4) represents the residual capacity of the system while it is working at time s. The random variable (5) defines the capacity loss between time points t and s. These random quantities are potentially useful for a preventive action. For example, if the total weight of working components is near k, then the operator may consider a maintenance procedure since the system may fail in a short time. On the other hand, if the capacity loss between two time points is larger than a fixed threshold, then the system may require maintenance. Furthermore, the random variable defined by (5) is useful to elicit information about capacity loss over disjoint time intervals.

Knowing the residual capacity of the system at a specific time point while the system is working is also important from a reliability economics point of view. For example, a power generation company may determine its investment strategy for its particular generation system depending on the value of $h(s) = E(W_n(s) | T > s)$ which provides information about the mean capacity of the system at a future time point s. The value of $h(s)$ together with the expected value of consumption at time s are important for company's future strategies.

If in particular $\omega_i = 1$ for all $i = 1, \ldots, n$, i.e. all components have the same weight, then the random variable (4) defines the number of working components while the system is working at time t, and the random variable (5) is the number of failed components between t and s while the system is working at time s, $t < s$. Therefore, these conditional random variables are of special importance also for understanding the dynamic behavior of usual k-out-of-n:G systems.

The residual capacity of the system after the failure of the system can be defined as

$$W_n(T) = \sum_{i=1}^{n} \omega_i I(T_i > T). \tag{6}$$

The random variable $W_n(T)$ is a useful quantity, and gives an idea of how much capacity should be available to replace the system.

In this paper, we study the distribution and mean of the conditional random variables defined by (4) and (5), and the mean of the random variable defined by (6). In Section 2, we obtain recursive equations for these characteristics under the general case when the components have different failure time distributions. Section 3 is devoted to the case when the system consists of two different types of components and we obtain non-recursive equations. Finally, in Section 4, we provide illustrative computational results.

2. The general case

Let $T_1, \ldots, T_n$ represent the lifetimes of n independent components having continuous lifetime distributions $F_i(t) = P\{T_i \leq t\}, i = 1, \ldots, n$. The two dimensional distribution of the stochastic process $W_n(t)$ obeys the following set of recurrences. For $n > 1$ and $m_1 \geq m_2$, considering the failure time of component n, it can be shown that

$$\begin{aligned} &P\{W_n(t) = m_1, W_n(s) = m_2\} \\ &\quad = P\{W_{n-1}(t) = m_1 - \omega_n, W_{n-1}(s) = m_2 - \omega_n\}\overline{F}_n(s) \\ &\qquad + P\{W_{n-1}(t) = m_1 - \omega_n, W_{n-1}(s) = m_2\}(F_n(s) - F_n(t)) \\ &\qquad + P\{W_{n-1}(t) = m_1, W_{n-1}(s) = m_2\}F_n(t) \end{aligned} \tag{7}$$

with initial conditions

$$P\{W_1(t) = m_1, W_1(s) = m_2\} = \begin{cases} \overline{F}_1(s) & \text{if } m_1 = m_2 = \omega_1 \\ F_1(s) - F_1(t) & \text{if } m_1 = \omega_1 \text{ and } m_2 = 0 \\ F_1(t) & \text{if } m_1 = 0 \text{ and } m_2 = 0 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

$t < s$.

Thus the distribution of the conditional random variable (5) can be computed from

$$P\{W_n(t) - W_n(s) = a | T > s\} = \frac{P\{W_n(t) - W_n(s) = a, T > s\}}{P\{T > s\}}. \tag{9}$$

By the definition of the system

$$\begin{aligned} P\{W_n(t) - W_n(s) = a, T > s\} &= P\{W_n(t) - W_n(s) = a, W_n(s) \geq k\} \\ &= \sum_{m_2 = k}^{\sum_{i=1}^{n} \omega_i} P\{W_n(t) - W_n(s) = a, W_n(s) = m_2\} \\ &= \sum_{m_2 = k}^{\sum_{i=1}^{n} \omega_i} P\{W_n(t) = a + m_2, W_n(s) = m_2\}. \end{aligned} \tag{10}$$

Therefore,

$$\begin{aligned} &P\{W_n(t) - W_n(s) = a | T > s\} \\ &\quad = \frac{1}{P\{T > s\}} \sum_{m_2 = k}^{\sum_{i=1}^{n} \omega_i} P\{W_n(t) = a + m_2, W_n(s) = m_2\}. \end{aligned} \tag{11}$$

The mean capacity loss between t and s can now be computed from

$$m(t,s) = E(W_n(t) - W_n(s) | T > s) = \sum_{a=0}^{\sum_{i=1}^{n} \omega_i - k} aP\{W_n(t) - W_n(s) = a | T > s\}. \tag{12}$$