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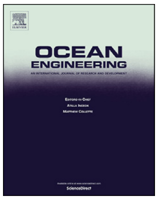

Analytic study on long wave transformation over a seamount with a pit

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ABSTRACT

In this paper, an analytic solution is derived for linear long waves scattering over a submarine seamount landform with a pit. The seamount is axisymmetric with a pit on the top. The water depth is defined by a trinomial function in the radial direction. The governing linear shallow water equation for long waves is expressed in the polar coordination, which is solved through separation of variables. As the topography is axisymmetric, solutions can be written as Fourier-cosine series. Waves over the seamount are expressed using Frobenius series expansion, while the water surface elevation in the outer region is expressed as Fourier-Bessel series, and the final solution is obtained by matching them at the conjunction. The solution can be degenerated into the previous analytic solutions for waves propagation over an axisymmetric pit or a submerged hump by adjusting the topography parameters.

1. Introduction

The earthquake in the Sumatra Sea, northern Indonesia on December 26, 2004 triggered a strong tsunami that caused tremendous casualties and property damage in Southeast Asia and South Asian countries (Synolakis and Bernard, 2006). Since then, the study of long-wave (e.g. tsunami wave) propagation in shallow water attracts more attention. Tsunami arrival time and run-up are the most important information for disaster prevention and reduction, but they are highly relevant to the propagation paths that feature with the scattering of submarine topographies (Satake, 1988). The ocean ridges, seamounts and the continental shelf may all act as natural guides for the propagation of long waves (such as tsunami) far away from its source (Titov et al., 2005).

Seamounts are one of the most common geomorphic features of the seabed. To understand the focusing and scattering phenomena of tsunamis or other long waves on the seamount is of practical significance for real-time tsunami prediction and coastal protection. Related to this, a series of research results obtained using different methods have been reported for wave transformation caused by three-dimensional topographies, such as islands or submerge shoals. The earliest study may be traced back to Homma (1950), who resolved a vertical cylindrical islands mounted on a parabolic shoal. The results showed that, for a particular incident wave frequency, the wave amplitude was unusually large on the shoal. Vastano and Reid (1967) validated Homma's results using a finite difference numerical model on a truncated domain. Longuet-Higgins (1967) presented in-depth work on the energy-trapping phenomenon of underwater axisymmetric topography. He considered a case of wave propagation over a submerged circular cylinder and mathematically proved that wave energy cannot be fully captured by a submerged circular sill and certain amount of energy will leak to infinity. As the leaked energy is extremely small for certain specific frequencies, the energy is nearly trapped in the vicinity of the topography. Such a near-trapping phenomenon corresponds to a large response from the topography, which is described as near-resonance. Barnard et al. (1983) conducted experiments to examine possible trapping of surface waves by a submerged circular sill. They observed that resonance is activated at certain locations over the sill and the wave amplitude could be amplified 4 to 5 times. Chamberlain and Porter (1999) adopted the modified mild-slope equation to approximate the scattering and near-trapping of surface waves by axisymmetric submerged circular shoals of variable water depth. Their results proved that, for each mode, near-trapping phenomena may be associated with a set of discrete frequencies and the degree of resonance response to the trapping is closely related to the topography. Energy leakage may also affect the pattern of long-distance wave propagation. The transoceanic tsunami caused by the 2006 Kuril Islands earthquake (M8.3) propagated across the entire Pacific Ocean. Tidal gauge records along the Pacific coast of Japan show that the maximum tsunami wave arrived more than 5 h later than the primary wave. The

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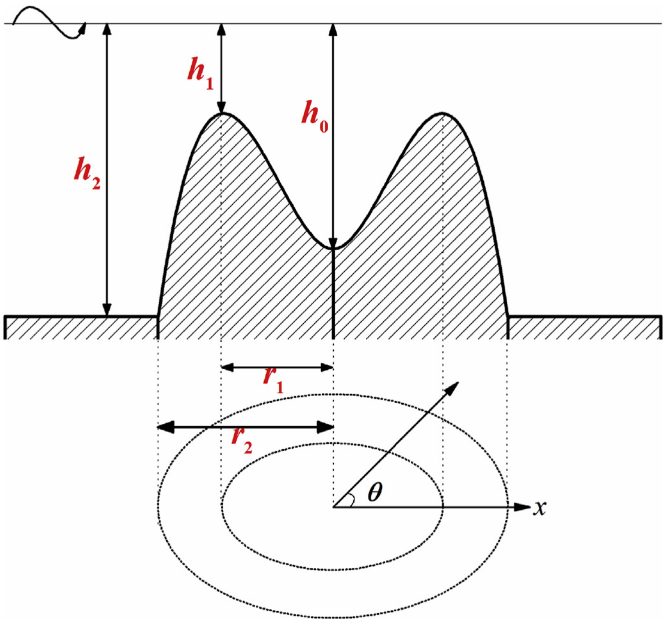


Fig. 1. Definition sketch of the seamount with a pit.

simulation results reported by Koshimura et al. (2008) indicated that the possible reason may be due to wave scattering caused by the Emperor seamounts.

For a more complex bottom geometry, Zhu and Zhang (1996) derived analytical solutions for the propagation of long surface waves around a conical island and over a paraboloidal shoal, where the paraboloid shoal is described as $h\ (r) = a\ r^2$ (r is the radial distance and the a is a topography parameter). Lin (2007) presented an analytic solution of the gentle slope equation for wave scattering and trapping around a truncated paraboloid shoal defined as $h\ (r) = a\ r^2$ (where r is the radial distance and the a is a topography parameter). Liu and Li (2007) sought an analytical solution for wave scattering by a submerged circular truncated shoal with the bottom geometry being an arbitrary power function $h(r) = ar^m$. Zhu and Harun (2009) further derived an analytic solution of long-wave over

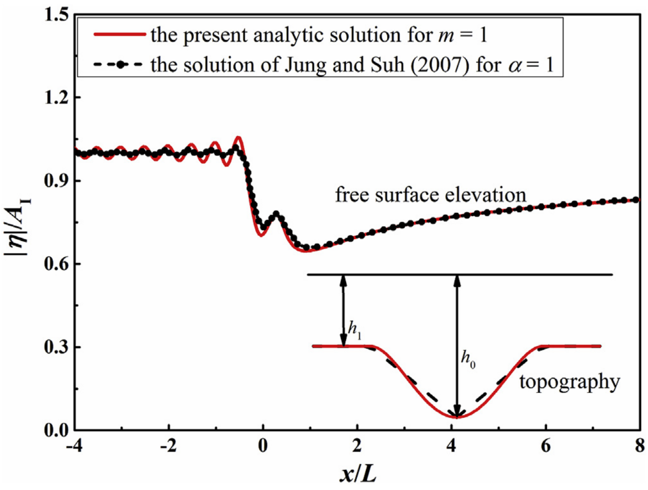
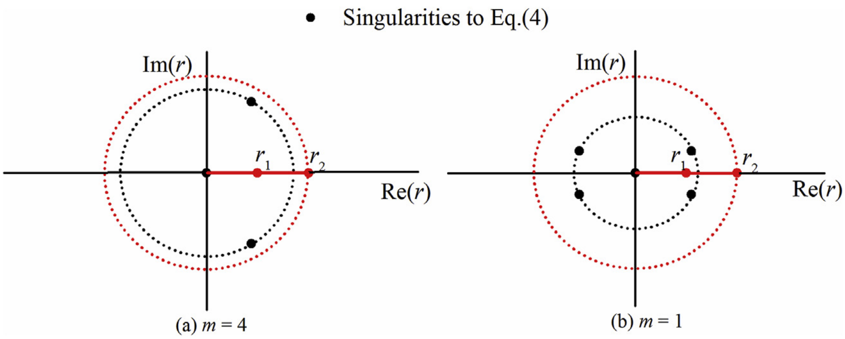


Fig. 4. Comparison between the present analytic solution and results of Jung and Suh (2007).



(a) $m = 4$ (b) $m = 1$

Fig. 2. Singularities to the Eq. (4).

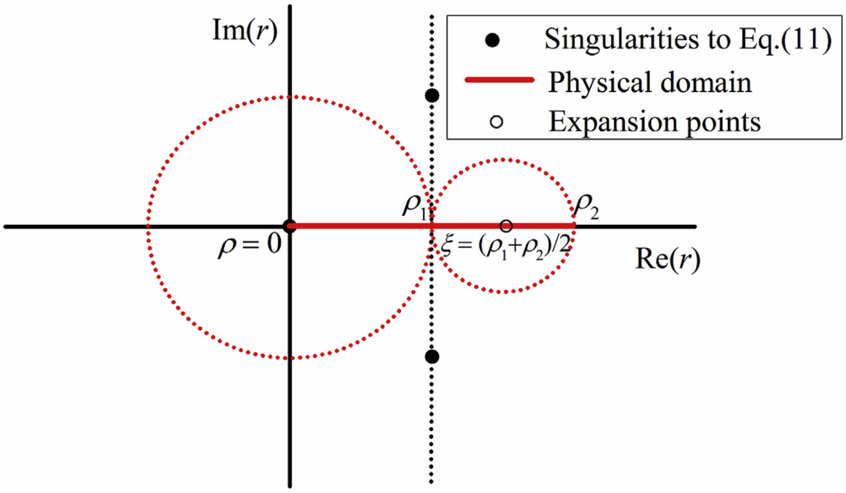


Fig. 3. Singularities to the Eq. (7), expansion points and convergent regions.