Ocean Engineering 154 (2018) 357–366

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Stochastic optimal control of a heave point wave energy converter based on a modified LQG approach

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ARTICLE INFO

Keywords:
Wave energy
Heave absorber
Modified LQG control
Displacement and control force restriction
Kalman filtration

ABSTRACT

The optimal control constrain problem of a wave energy point absorber is constrained due to limited stroke and saturation of the control force actuator. The basic idea of this paper is to control the motion of the absorber by a modified LQG control where the constraints on the displacement and actuator force are approximately considered by counteracting the absorbed power in the objective quadratic functional. Based on rational approximations to the radiation force and the wave load, the integrated dynamic system can be reformulated as a linear stochastic differential equation which is driven by a unit intensity Gaussian white noise. The optimal LQG control force becomes a linear function of the state vector of the integrated system, which can only be partially observed. In order to remedy this problem, the control is combined with a Kalman filter observer. The obtained sub-optimal solution has been compared to the numerical optimal solution obtained by nonlinear programming. With suitable calculated gain parameters, the LQG controller can provide approximately the amount of the averaged absorbed power as that of the numerical optimum.

1. Introduction

A wave energy converter (WEC) extracts the mechanical energy in the wave motion and converts it into electric energy. Different kinds of WEC devices have been developed such as the oscillating water column plant (Ozkop and Altas, 2017), overtopping types like the Wave Dragon (Wavedragon, 2005), the Pelamis (Pelamis Wave, 2012), Archimedes Wave Swing (Archimedes Wave Swing, 2004), and the Wave Star Energy plant (Wave Star Energy, 2003).

A wave energy point absorber is a wave energy converter (WEC) with horizontal dimensions significantly smaller than the dominating wave length, which is capable of absorbing energy from waves propagating in arbitrary directions. Especially, a heave absorber is constrained by a mooring system or otherwise to move merely in the vertical direction.

Significant increase of the power take-off (PTO) of a heave absorber may be achieved by using an active vibration control of the vertical motion (Ringwood et al., 2014). In this connection many control strategies typical of the proportional derivative (PD) type have been suggested in the literatures. Nielsen et al. (2013) derived the optimal control law in irregular sea-states for a heave point absorber with non-linear buoyancy in case of no constraints on the displacements and the control force. The optimal control force turns out to make the absorber maximal flexible by eliminating the inertial load and the buoyancy stiffness totally. Further, the control law has feed-back from the present displacement and acceleration of the absorber and a non-causal feedback from the future velocities. Hence, for practical applications the indicated control law requires a prediction of future velocities. The predictor introduces uncertainty in the problem and makes the control sub-optimal.

Generally, there are constraints on the motion of the absorber due to the limited stroke of the actuator of the control system. Similarly, the available control force will be constrained between certain limits due to saturation. Based on the optimal control for the unconstrained case, Sichani et al. (2014) proposed an extension to the unconstrained case, where the displacement were achieved by adding the nonlinear artificial springs to the buoyancy, which were achieved close to the boundaries. Using predictive PD control, Wang et al. (2015) analyzed the motions of a point absorber. Based on truncated Fourier series of the control force and the velocity, the problem is converted to an optimization problem with a convex quadratic objective functional and nonlinear constraints. Cretel et al. (2011) proposed a control scheme to maximize the absorbed energy by a wave energy point absorber based on model predictive control. As a result of the introduction of triangle-hold discretization approach where the control force and the wave load need to be continuous piecewise linear, the objective functional is reformulated as a convex quadratic

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https://doi.org/10.1016/j.oceaneng.2018.02.021
Received 1 May 2017; Received in revised form 3 January 2018; Accepted 5 February 2018

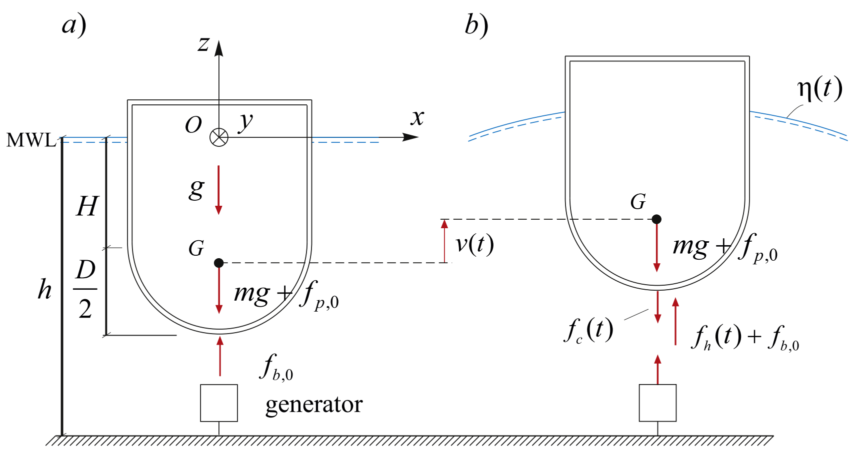

Fig. 1. Loads on heave absorber. a) Static equilibrium state. b) Dynamic state.

function of the increment of the control force. Constraints on the displacement of the absorber and the control force can be enforced to the system by affine inequality constraints on the input increment control force. However, the control law may give rise to feasibility issues for the hard constraints on the control force and may cause large amounts of energy flowing in and out of the system. Especially, it turns out that the instantaneous absorbed power may undergo large negative excursions which is not the care for the optimal control. Li et al. (2012) analyzed the nearly optimal control of wave energy converter with the state and control input constraints based on Pontryagins Minimum Principle. Further, the interior penalty term included in the cost functional replaces the state constraints, preventing the optimal state trajectory from approaching the boundary of the permitted region. The nearly optimal control approximated using discretization and dynamic programming turns out to be bang-bang control on the condition that the portions of the singular arc assuming that the times in which this happens are negligible are ignored, without rigorous proof for that. Zou et al. (2017) demonstrated that the singular arc part of the optimal control cannot be neglected and significant portions of time may become on singular arcs depending on the initial conditions and on the maximum control level. Hartl et al. (1995) presented the Pontryagins maximum principle for optimal control problems with both pure state and mixed variables inequality constraints. Further, the mixed constraints are the constraints on control variables that may depend on the state variables and the time.

As the optimal control turns out to be noncausal, i.e. the control law depends on the future motion of the absorber or wave load, prediction of the motion of the absorber or wave load should be considered. To remedy this question, a causal closed-loop controller with the feedback information is proposed. In case of infinite control horizon this problem can be circumvented by LQG control. Lattanzio and Scruggs (2011) derived the optimal causal controller for wave energy converter and the determination of the optimal causal controller distills to a nonstandard LQG optimal control problem, which can be solved easily. Scruggs et al. (2013) formulated the LQG control problem for wave energy converter and compared the results with the optimal noncausal control through choosing proper weights. Kassem et al. (2015) maximized the take-off power from a two-body point absorber with a mooring wave energy converter based on LQG approach and demonstrated the feasibility and effectiveness of the LQG control. In the present paper the basic idea is to deal with stochastic optimal control of a heave point wave absorber with constraints on the displacement and the control force, and with noisy observation on the displacement and the velocity. The radiation force and the wave load are reformulated as output of rational approximate filters. The integrated dynamic system may be given by a linear stochastic state vector differential equation driven by a Gaussian white noise. The idea of the paper is to take the constraints on the displacement and the control force into consideration by introducing negative penalty terms of the two parameters in the Lagrangian of a LQG approach, where the weights are calibrated against a nonlinear programming solution to provide the same mean power take-off. This does not guarantee a local observation of the indicated constraints, but merely that these are fulfilled in average. Further, the controller is combined with a Kalman filter, for which the reason is merely that the displacement and the velocity can be observed. The obtained sub-optimal results from assumed full state observation and partial state observation will be discussed and compared to numerical optimal controller from nonlinear programming. The stochasticity of the control problem origins partially from the nonobservable wave load and the noise related to the measured displacements and velocities. Hence, the indicated quantities need to be modelled by stochastic process.

2. Equation of motion of point absorber

Although, only the heave absorber shown in Fig. 1 will be analyzed, all results, including the equation of motion and control laws, may easily be carried over to other single-degree-of-freedom systems by slight modifications. The (x, y, z)-coordinate system is introduced as shown in Fig. 1. The original O is placed in the mean water level (MWL) at the centerline of the point absorber. The x-axis is the horizonal orientation in the direction of the wave propagation, and the z-axis is vertical orientation in the upward direction. Only two-dimensional (plane) regular or irregular waves are considered. The motion $v(t)$ of the body in the vertical z direction is defined relative to the static equilibrium state, where the static buoyancy force balances the gravity force and a possible static pre-stressing force from the generator.

In the dynamic state caused by the surface elevation $\eta(t)$ the WEC is excited by dynamic hydrodynamic force, $f_h(t)$, in addition to the static buoyancy force, and by an additional control force, $f_c(t)$ from an external hydraulic or electric force generator as the PTO system, which is used to control the motion of the absorber and to achieve maximal wave energy absorption. In theoretical research, it's assumed that the PTO system can provide the reactive power. In applications, the cylinder can operate as a pump, producing a bi-directional flow, which drives a hydraulic motor. The motor adapts to the flow and rectifies the flow into a unidirectional turning of the generator. Further, The PTO system will absorb a positive power from the absorber if the control force $f_c(t)$ and the velocity $\dot{v}(t)$ are in counter phase. In opposite case, the PTO system acts as a motor and supplies energy to the absorber. Next, the mechanical energy stored in the absorber is converted into electrical energy via an generator. Henceforth, $f_c(t)$ considered positive in the opposite direction of $v(t)$ will be referred to as the control force. Then, the equation of motion becomes:

$$m\ddot{v}(t) = f_h(t) - f_c(t) \tag{1}$$

Assuming linear wave theory $f_h(t)$ may be written as a superposition of the following contributions: