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Cold-standby sequencing optimization considering mission cost

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ABSTRACT

This paper considers the optimal standby component sequencing problem (SESP) for 1-out-of-*N*: G heterogeneous cold-standby systems. Given the desired cold-standby redundancy level and a fixed set of components, the objective of the optimal system operation scheduling is to select the initiation sequence of the system components so as to minimize the expected mission cost of the system. Based on a discrete approximation of time-to-failure distributions of the system components, the mission reliability and expected mission cost are simultaneously evaluated using the universal generating function technique. A genetic algorithm is used as an optimization tool for solving the formulated SESP problem for the 1-out-of-*N*: G heterogeneous cold-standby systems. Several examples are given to illustrate the considered problem and the proposed solution methodology.

1. Introduction

Using standby redundancy to improve the reliability of a system is a well-known principle in the reliability engineering field. Usually standby systems are designed from physically identical components. However, during the system exploitation the replacement and/or repairs can lead to differences in components' statistical behavior. Moreover, as different system components can have different physical locations, they can work in different conditions which cause differences in their time-to-failure distributions. Thus the problem of optimizing the operation of heterogeneous cold-standby systems arises.

System reliability optimization problems such as redundancy allocation and reliability allocation problems are proven to be NP-hard [1]. Many solution methodologies have been proposed to solve them [2–4]. This paper particularly focuses on the optimal standby component sequencing problem (SESP) of 1-out-of-*N*: G heterogeneous cold-standby systems with an objective to minimize the expected system mission cost associated with energy consumption, lubrication, cooling, startup expenses etc.

Exact optimization methods like dynamic programming [5], Lagrangean multipliers [6], and integer programming [7], have been proposed for solving the redundancy allocation problem (RAP) of 1-out-of-*N*: G series-parallel systems with hot standby redundancy and homogeneous backup scheme where one type of components can be substituted only by the same type of components to achieve fault tolerance. Later on meta-heuristic optimization methods like genetic algorithm, ant colony optimization algorithm, the improved surrogate constraint method, and Tabu search [8–11] were proposed to solve the RAP of 1-out-of-*N*: G and *K*-out-of-*N*: G series-parallel systems with hot standby redundancy and adapted heterogeneous backup scheme where one type of components can be substituted with a different type of functionally-equivalent components to achieve fault tolerance. In [12], a solution methodology based on integer programming was proposed to determine the optimal design configuration for non-repairable 1-out-of-*N*: G series-parallel systems with cold standby redundancy. In [13–15], a solution methodology was proposed for solving the RAP of 1-out-of-*N*: G and *K*-out-of-*N*: G heterogeneous series-parallel systems where standby subsystems exclusively involve either hot or cold standby redundancy.

However, none of the aforementioned approaches solved the SESP of standby systems considering the mission cost, which is however a critical factor for the system design and performance. Recently Ref. [16] considered the redundancy allocation of standby systems subject to cost and energy consumption constraints, where the energy consumption of the system is simply modeled as the sum of energy consumption of all the selected component choices for the system design without considering components' real life times. Such model is not accurate as the energy consumption and the total mission cost should be functions of actual operation times of the system components. In the standby systems, the operational time of a component is dynamic/random. Thus, the total mission cost of the standby system exhibits random behavior given the constant startup and per time unit exploitation

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Acronmys

cdf	cumulative distribution function
pdf	probability density function
pmf	probability mass function
r.v.	random variable
GA	genetic algorithm
UGF	universal generation function
SESP	standby component sequencing problem
RAP	redundancy allocation problem

Nomenclature

N	number of components in the system
$s(i)$	index of the component initiated after $i-1$ failures
T_j	*r.v.* representing the time-to-failure of component j
$t_{j,h}$	hth realization of T_j
$p_{j,h}$	$\Pr\{T_j=t_{j,h}\}$
w_j	cost (per time unit) of running component j
v_j	startup cost of component j
τ	mission time
R	system reliability
E	expected mission cost
X_i	random cumulative working time of components $s(1)$, $s(2)$, …, $s(i)$: $X_i=\min\left\{\tau, \sum_{j=1}^{i} T_{s(j)}\right\}$
$x_{j,f}$	fth realization of X_j
$Q_{j,f}$	$\Pr\{X_j=x_{j,f}\}$
M_i	number of different realizations of X_i
K_j	number of different realizations of T_j
$u_j(z)$	u-function representing discrete distribution of T_j
$U_i(z)$	u-function representing discrete distribution of X_i
m	number of considered mission time intervals
Δ	duration of each time interval

costs of the components. To the best of our knowledge, there has been no work done on the practical modeling of the mission cost of a standby system with dynamic/random component failure behaviors. In this paper, we propose a novel numerical method for the mission cost analysis of cold-standby systems. Based on the proposed mission cost model, this work further considers the problem of optimal sequencing of the components in 1-out-of-N: G heterogeneous cold-standby systems with the objective to minimize the expected mission cost of the system.

The presented method for evaluating the mission reliability and expected mission cost is based on a discrete approximation of time-to-failure distributions of system components. Using the approximated discrete distributions allows one to obtain close estimates of the mission characteristics and to choose the optimal initiation sequence of the system components.

The remainder of the paper is organized as follows. Section 2 describes the system model and assumptions. Section 3 presents an algorithm for evaluating the mission cost and reliability of a 1-out-of-N: G heterogeneous cold-standby system. Section 4 contains examples for illustrating the proposed method. Section 5 presents the optimization algorithm for sequencing the components in a cold standby system. The optimization results for an example system are given in Section 6. Section 7 presents conclusions and directions for the future work.

2. The system model

The system consists of N non-identical components with given time-to-failure distributions. One component is put in the operation at the beginning of the mission and the remaining $N-1$ components wait in the cold standby mode before being put into operation. One working component can successfully accomplish the system mission, i.e. the system is 1-out-of-N: G. The overall mission fails if all components fail before the mission time.

The system reliability does not depend on the initiation sequence of components as it is equal to the probability that the sum of all the components' life times is not less than the mission time. However the actual operation time of each component strongly depends on the components initiation sequence. Therefore, when the components' operation costs are different, the entire expected mission cost also depends on this sequence. Thus, without changing the overall system reliability one can reduce the expected mission cost by the optimal sequencing of the standby components. The following Section 3 presents an algorithm for evaluating the reliability and the expected mission cost for an arbitrary sequence of the standby components.

The following assumptions are made:

(1) The set of components is fixed.
(2) The time-to-failure distributions of components are independent.
(3) The characteristics of the components do not change during the mission.
(4) The standby components are put in operation in a predetermined order that cannot be changed during the mission.
(5) The mission time is fixed.
(6) Compared to the mission time the restoration times are negligible.
(7) The fault detection and component replacement/switching mechanisms are fully reliable.

3. Mission reliability and cost evaluation algorithm

We divide the mission time τ into m equal intervals with duration $\Delta=\tau/m$. Having the expression for the failure time distribution of any component j in the form of its *cdf* we can obtain the probability $p_{j,h}$ that component j fails in the hth time interval (time between $\Delta(h-1)$ and Δh). For example, for the exponentially distributed failure time with failure rate λ_j:

$$p_{j,h}=[\exp(\lambda_j\Delta)-1]\exp(-\lambda_j\Delta h); \quad (1)$$

for the Weibull distribution with parameters η and β

$$p_{j,h}=\exp\{-[\Delta(h-1)/\eta]^{\beta}\}-\exp\{-[\Delta h/\eta]^{\beta}\}. \quad (2)$$

Instead of considering the random continuous time-to-failure of component j we consider the random discrete time-to-failure T_j of this component assuming that the *pmf* of T_j presented in the form of pairs $(t_{j,h}=\Delta h,\ p_{j,h}=\Pr\{T_j=t_{j,h}\})$ for $1\le h\le m$ approximates the *pdf* of the component's time-to-failure.

As no component should work longer than the mission time τ the probability that the component does not fail during the mission $p_{j,m}=\Pr\{T_j\ge\tau\}$ is determined as $p_{j,m}=1-\sum_{h=1}^{m-1}p_{j,h}$.