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# Analytical solution and simplified formula for earthquake induced hydrodynamic pressure on elliptical hollow cylinders in water

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ABSTRACT

Analytical expressions for calculating the hydrodynamic pressure on an elliptical hollow cylinder caused by outer and inner water under earthquake are obtained. Based on the radiation theory, the analytical solutions of hydrodynamic pressure and hydrodynamic force on elliptical hollow cylinder are accurately derived by assigning reasonable boundary conditions and by solving the Mathieu's differential equation in elliptical coordinate system. Added mass expressions distributed over the height of the cylinder are used to represent the hydrodynamic interaction effects, which are extracted from the hydrodynamic force expressions. These added mass expressions are so complicated that lead to their value difficult to be calculated. Therefore, these added mass expressions are dealt with dimensionless method and are simplified by curve fitting method in this study. The simplified formulas are only relevant to three dimensionless parameters including width-depth ratio, ratio of long to short axis of the ellipse and relative distance above base. The calculation results show that the simplified formulas are in good agreement with the analytical solutions.

## 1. Introduction

Offshore structures may be subjected to significant hydrodynamic pressures during earthquakes, which influences the dynamic response and dynamic properties of these structures, such as intake towers, oil storage tanks and bridge piers. Therefore, to design an offshore structure, the evaluation of the hydrodynamic pressure on the structure is an important task.

The estimation of hydrodynamic pressure on a circular cylinder has been studied by many researchers. Liaw and Chopra (1974) investigated the significance of hydrodynamic pressures on the earthquake response of cantilever circular cylinders. The authors discovered that water compressibility was negligible for slender cylinders but important for squat cylinders vibrating at high frequency, and the surface waves was significant only at very low frequencies. Moreover, the effect of the hydrodynamic pressure can be conveniently modeled as an 'added mass' when surface wave and water compressibility are ignored. Williams (1986) utilized the boundary integral method to calculate the dynamic response of circular cylinders subjected to high frequency horizontal ground excitation. Tanaka and Hudspeth (1988) presented an eigenfunction for the dynamic response of circular cylinders to earthquake excitation in compressible water. Han and Xu (1996) presented a theoretical model of an added mass representation for a flexible circular cylinder vibrating in water and presented a simple formula for evaluating the natural frequencies using the added mass representation. Chen (1997) developed a finite-difference scheme to solve nonlinear hydrodynamic pressures acting on a circular cylinder during earthquakes. Liu et al. (2008) presented hydrodynamic pressure expressions for circular hollow bridge piers caused by outer and inner water, which were modified and simplified by Li and Yang (2013) in some extent. Du et al. (2014) proposed a simplified formula of hydrodynamic force on circular cylinders in the time domain based on the analytical solution, which is expressed in the form of added mass in low frequency vibration and in the form of added mass and added damping in high frequency vibration. Yang and Li (2013a) proposed the expanded Morison equation to calculate the hydrodynamic force of hollow circular piers caused by inner water.

Some researchers have also investigated the earthquake-induced hydrodynamic pressure on an axisymmetric cylinder. Liaw and Chopra (1975) used the finite element method to analyze the response of axisymmetric tower partly submerged in water subjected to earthquake ground motion. Sun and Nogami (1991) proposed a semi-analytical and semi-numerical approach for axisymmetric cylinders, which can model accurately the water compressibility and gravity waves on the water surface. Park et al. (1991) utilized the finite element technique incorporating the infinite element to evaluate hydrodynamic pressure on

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axisymmetric cylinders. Avilés and Li (2001) proposed a boundary method to evaluate the hydrodynamic pressure on rigid axisymmetric cylinders in the compressible water on the flexible bedrock. In addition, Goyal and Chopra (1989) developed a simplified procedure to evaluate the added mass for intake tower with two-symmetric-axes cross section; Yang and Li (2013b) presented a new added mass method to calculate hydrodynamic pressure on piers with arbitrary cross-section, which was based on the fundamental frequency reduction rate of piers.

In this paper, we investigate earthquake induced hydrodynamic pressure on elliptical hollow cylinders caused by outer water and inner water. The elliptical coordinates are used by Williams (1985) and Bhatta (2005) to deduce the analytical expressions for the wave force on vertical cylinders with elliptical cross section. To derive the analytical solution for the earthquake induced hydrodynamic force on elliptical hollow cylinders, we also use the elliptical coordinates. In case of the ground motion along the major axis and along the minor axis the ellipse, these analytical solutions are obtained in terms of circumferential Mathieu functions and radial Mathieu functions by using the method of separation of variables. However, those analytical solutions are not suitable for engineering application because their expressions are extremely complicated. We use the self-made programs based on special math software MATLAB to calculate the value of those expressions. Furthermore, those complicated analytical expressions are simplified by curve fitting method. Comparing the simplified formula with the analytical solution, the results show that the simplified formula is accurate and concise enough for engineering applications.

## 2. Differential equation and boundary condition

The system of the interaction between the vertical cylinder and water is shown in Fig. 1. The vertical cylinder is simplified as an elliptical hollow cylinder with the bottom fixed at the rigid ground. Water is assumed to be incompressible and inviscid. As shown in Fig. 1, $a$ and $b$ are outside semi-major and semi-minor axes, $a_1$ and $b_1$ are inside semi-major and semi-minor axes of the elliptic cross section, and the water depth is $h$. The water is treated as calm before the action of earthquake and only the radiation wave would be simulated under earthquake. A harmonic wave with frequency $\omega$ is considered here as the seismic excitation and the displacement response of the cylinder can be expressed as $u(t) = Ue^{i\omega t}$. Here, $U$ is the amplitude, $t$ is time. Frequency $\omega$ is considered here as the seismic excitation and the displacement response of the cylinder can be expressed as $u(t)$. Here, $U$ is the amplitude, $t$ is time.

It is a property of linear time invariant systems that when the excitation is a simple harmonic motion, steady-state response is also a simple harmonic motion at the same frequency (Liaw and Chopra, 1974). Therefore, the small amplitude irrotational motion is governed by the Laplace's equation in Cartesian coordinate system, which can be expressed as

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = 0 \tag{1}$$

where $p(x, y, z, t)$ denotes the hydrodynamic pressure.

The elliptic coordinate system is depicted in Fig. 2. The transformation from Cartesian coordinate to elliptical coordinate is

$$x = \mu \cosh \xi \cos \eta \tag{2a}$$

$$y = \mu \sinh \xi \sin \eta \tag{2b}$$

where $\mu = \sqrt{a^2 - b^2}$; $\xi$ and $\eta$ are orthogonally intersecting families of confocal ellipses and hyperbolas, respectively. The coordinate of the surface of the elliptical cylinder is expressed by

$$\xi = \xi_0 = \tan^{-1}(\tilde{b}/\tilde{a}) \tag{3}$$

where $\tilde{a} = a$ and $\tilde{b} = b$ for outer water, $\tilde{a} = a_1$ and $\tilde{b} = b_1$ for inner water. The coordinate $\xi = 0$ express the straight line of length.

Thus, Eq. (1) can be rewritten as

$$\frac{1}{\mu^2(\cosh^2 \xi - \cos^2 \eta)}\left(\frac{\partial^2 p}{\partial \xi^2} + \frac{\partial^2 p}{\partial \eta^2}\right) + \frac{\partial^2 p}{\partial z^2} = 0 \tag{4}$$

which is equivalent to

$$\frac{2}{\mu^2(\cosh 2\xi - \cos 2\eta)}\left(\frac{\partial^2 p}{\partial \xi^2} + \frac{\partial^2 p}{\partial \eta^2}\right) + \frac{\partial^2 p}{\partial z^2} = 0 \tag{5}$$

According to Liaw and Chopra (1974), the effects of surrounding water can be entirely equivalent to an added mass when water is incompressible. In reality, the cylinder is flexible. Li and Yang (2013) investigated the added mass caused by elastic vibration and rigid motion of the flexible circular cylinder. The results indicated that the added mass caused by elastic vibration is approximately equal to added mass caused by rigid motion of the cylinder. Thus, the elliptical cylinder is assumed to be rigid in this study.

According to Li and Yang (2013), free surface wave with higher frequency has less effect on the hydrodynamic force on cylinders in deep water, no matter the hydrodynamic force is caused by outer water or inner water. Namely, free surface wave could be ignored if the frequency is higher. In general, the earthquake frequency and the structural frequency are higher. Liaw and Chopra (1974) also found that free surface wave was of little consequence in the earthquake response of towers surrounded by water. Therefore, free surface waves are assumed to be not generated in this study.

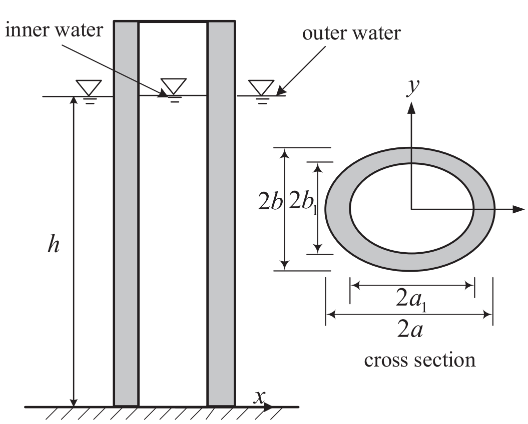


**Fig. 1.** Definition of the problem.

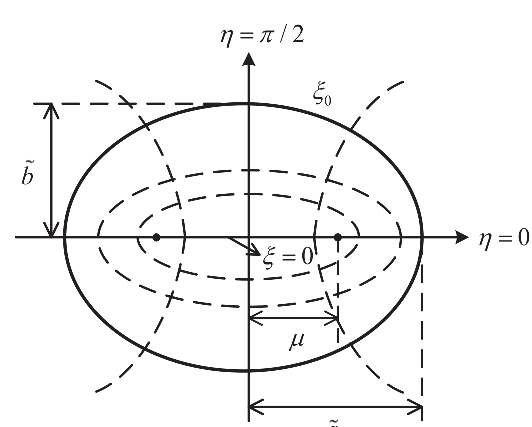


**Fig. 2.** Elliptic cylindrical coordinate.