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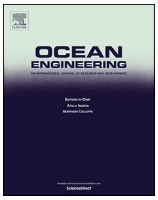

An efficient approach to forecast water levels owing to the interaction of tide and surge associated with a storm along the coast of Bangladesh

Gour Chandra Paul [a,*], Sukumar Senthilkumar [b,c,d], Rana Pria [e]

[a] Department of Mathematics, University of Rajshahi, Rajshahi 6205, Bangladesh
[b] School of Mathematical Sciences, Universiti Sains Malaysia, Pulau Pinang 11800, Malaysia
[c] Center for Advanced Image and Information Technology, Division of Computer Science and Engineering, School of Electronics & Information Engineering, Chon Buk National University, 664-14, 1Ga, Deok Jin-Dong, Jeonju, Chon Buk 561-756, South Korea
[d] School of Computing Science and Engineering, Vellore Institute of Technology University, Tamilnadu, Vellore 632014, India
[e] Mathematics Discipline, Khulna University, Khulna 9208, Bangladesh

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ABSTRACT

Innovative formulation of new algorithm(s) is still a great challenge in scientific research community for solving any real time non-linear problems. Based on the above notion, in this study, the method of lines (MOL) in addition with the newly introduced embedded RKARMS(4,4) technique is adapted for numerical prediction of water levels owing to the interaction of tide and surge associated with a cyclone. To perform the desired task, initially a transformation is carried out to convert the vertically integrated shallow water equations in Cartesian coordinates with boundary conditions into ordinary differential equations (ODEs) of initial valued, which are solved by the RKARMS(4,4) technique to attain results. The model is specifically designed for the coast of Bangladesh. To incorporate coastal complexities along the region of interest with minimum cost, one way nested models are used. A stable tidal condition is generated applying the M_2 tidal constituent along the southern open boundary of the parent model. The newly designed model is applied to compute water levels due to combined effect of tide and surge associated with the recent severe storm AILA along the coast of Bangladesh. Simulated results show the new approach performs well and ensures conformity with observation.

1. Introduction

The coastal region of Bangladesh is frequently assailed by surges associated with tropical storms and a severe devastation takes place during each storm (Paul and Ismail, 2012a). The reasons behind the devastation are that surge levels along the region are influenced by some well-known factors, such as complex land-sea interface, offshore islands, shallow bathymetry, huge discharge through the three major rivers Padma, Brahmaputra and Meghna and other rivers, etc. (Debsarma, 2009; Paul et al., 2014). Thus an effective model capable of forecasting water level accurately for the region is highly desirable (Paul and Ismail, 2013). Due to the scientific advancement and availability of information technology, some numerical models for this region have already been proved to be useful for disaster planning and coastal management and research is still being carried out for finding out more accurate method towards the same (Paul et al., 2014). However, designing of new innovative techniques still and forever plays major role in research and in its development to determine best solution for any real-time problem (Senthilkumar and Paul, 2012). From the literature point of view, it is seen that the MOL is a very powerful method for the treatment of boundary value problems (BVPs) arising in Mathematics, Engineering and Physical sciences and can be applied to all major classes of partial differential equations (PDEs). It has the advantages of less computational time, no relative convergence problems and numerical stability (Sun et al., 1993; Ismail et al., 2007; Paul et al., 2014). The method is more effective than the regular finite difference method (FDM) in terms of accuracy (Sadiku and Gorcia, 2000). Recently, emphasis is also given on the solution of shallow water equations (SWEs) with the use of the MOL. Ismail et al. (2007) solved SWEs using the MOL in simulating the Indonesian tsunami of 2004 along the coastal belt of Peninsular Malaysia and Thailand. Based on the study of Ismail et al. (2007), Paul et al. (2014) investigated storm surge problem for the first time using the MOL along the coast of Bangladesh, where they found the MOL to be a suitable tool to predict water level associated with a storm in an efficient way over the FDM with respect to the facts mentioned above. It is to be pointed out here that Paul et al. (2014) conducted their study using the MOL in addition with the well-established classical 4th order Runge-Kutta (RK(4,4)) method. According to them, one of the main advantages of the MOL over the FDM is that the efficiency of the results can be increased in coordination with some well-established ODE solvers. It is

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* Corresponding author.
E-mail addresses: pcgour2001@yahoo.com (G.C. Paul), ssenthilkumar1974@yahoo.co.in (S. Senthilkumar), ranapriapaul@yahoo.com (R. Pria).

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to be noted here that there are some difficulties in implementing the RK(4, 4) method in estimating numerical solution, one of which is the absence of error estimation procedure (Yaacob and Sanugi, 1998). Several methods have been developed to overcome these weaknesses; specifically this is done by introducing the procedure that can estimate the errors in the numerical results (Yaacob and Sanugi, 1998). Worth mentioning of the methods are due to Merson (1957) and Fehlberg (1969). Evans and Yaakub (1995) and Yaakub and Evans (1999) introduced an embedded RK(4,4) method, which is actually two different RK methods, namely RK arithmetic mean (RKAM) and RK contra harmonic mean (RKCoM) of the same order 4. Yaacob and Sanugi (1998) adapted a much simpler RK-embedded method of order 4 based on the harmonic mean for solving IVPs, where they were able to reduce computational cost for complicated problems.

Recently, a new fourth order embedded RKAHeM(4,4) method and algorithm based on RK arithmetic mean (RKAM(4,4)) and RK Heronien mean (RKHeM(4,4)) with error control were proposed by Senthilkumar (2009) to solve the real time application problems in image processing under Cellular Neural Network (CNN) environment. Ponalagusamy and Senthilkumar (2009) introduced a novel embedded fourth order RK root mean square (RKARMS(4,4)) method to investigate raster CNN simulation. This embedded method was developed using RKAM(4,4) and RK root men square (RKRMS(4,4)) methods. A detailed illustration related to local truncation error (LTE), global truncation error (GTE), error estimates (EEs) and its behavioural characteristics for controlling fourth order and four stage RK numerical algorithms is addressed in Senthilkumar (2009).

In this paper, we intend to solve SWEs in cartesian coordinates adapting the MOL in addition with the newly proposed RKARMS(4,4) method for the prediction of water levels due to the combined effect of tide and surge associated with a storm along the coast of Bangladesh. The purpose of the study is to test whether the efficiency of the results obtained in Paul et al. (2014) can be increased by the new approach adopted in this study. The present work will improve on that of Paul et al. (2014), who used a resolution of grids capable of incorporating only major offshore islands along the coast of Bangladesh.

The remainder of the paper is organized as follows. Section 2 deals with theoretical foundation. The data sources are presented in section 3. A short note on the RKARMS(4,4) technique and its LTE are addressed in section 4 including EE. In section 5, numerical procedures are discussed in detail. Simulated results, analysis, and model validation are presented in section 6 and the conclusion is given in section 7.

2. Theoretical foundation

As in Paul et al. (2014), in the formulation of the model, a system of rectangular Cartesian coordinates is used in which the origin, *O*, is set at the undisturbed level of the sea surface. *OX* and *OY* point towards the south and east, respectively, and *OZ* is directed vertically upwards. The displaced position of the sea surface is given by $z = \zeta(x, y, t)$ and the position of the sea floor by $z = -h(x, y)$, then the depth averaged equation of continuity can be given by

$$\frac{\partial \zeta}{\partial t} = -\frac{\partial}{\partial x}[(\zeta + h)u] - \frac{\partial}{\partial y}[(\zeta + h)v], \tag{1}$$

and the depth averaged x and y components of the momentum equation can, respectively, be given by

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} + fv - g\frac{\partial \zeta}{\partial x} + \frac{T_x}{\rho(\zeta + h)} - \frac{C_f u(u^2 + v^2)^{1/2}}{\zeta + h}, \tag{2}$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - fu - g\frac{\partial \zeta}{\partial y} + \frac{T_y}{\rho(\zeta + h)} - \frac{C_f v(u^2 + v^2)^{1/2}}{\zeta + h}, \tag{3}$$

where pressure is taken as hydrostatic and the sea bottom friction is parameterized in terms of a conventional quadratic law.

In the above equations, u and v represent the Reynolds averaged components of velocity in the directions of x and y, respectively; $f = 2\omega \sin \phi$ the Coriolis parameter, where ω ($= 7.29 \times 10^{-5}$ rad s^{-1}) is the angular speed of the Earth rotation and ϕ is the latitude, wherever necessary, of the place of interest; g the acceleration due to gravity; ρ ($= 1.03 \times 10^3$ kg m^{-3}) the density of the sea water, assumed to be homogeneous; C_f the bottom friction coefficient; (T_x, T_y) represents the components of wind stress.

In this study, surface stresses are parameterized by the conventional quadratic law (see Paul et al., 2014),

$$(T_x, T_y) = \rho_a C_D \left(u_a^2 + v_a^2\right)^{\frac{1}{2}} (u_a, v_a), \tag{4}$$

where C_D and ρ_a are the drag coefficient and air density, respectively, and u_a and v_a are the respective x and y components of the surface wind. In our study, we used $C_D = 0.0028$, a value used by most modelers. Following Paul et al. (2014), we have used the empirical formula due to Jelesnianski (1965) for the surface wind field over the model domain and is given by

$$V_a = \begin{cases} V_0\sqrt{(r_a/R)^3} & \text{for all } r_a \le R \\ V_0\sqrt{(R/r_a)} & \text{for all } r_a > R \end{cases} \tag{5}$$

where V_0 is the maximum sustained wind at the radial distance R from the eye of the cyclone and r_a is any radial distance from the eye at which the wind field is desired. It is to be noted here that there are various formulae for generating surface wind field, but the reason behind the use of the above formula can be found in Paul and Ismail (2012a, b).

2.1. Boundary conditions

The boundaries of the model fall into two categories, namely closed boundaries, representing coastal and island boundaries, and open boundaries. The component of current along the outward directed normal to the coastal and island boundaries is considered as zero and for open boundaries, a condition of radiation type is used for the coarser grid model to allow the disturbance, generated within the model area, to go out through the open boundary (see Paul and Ismail, 2013). It is to be noted here that the boundaries of our analysis area are treated as straight lines in the open sea. Following Roy (1995), the western, eastern and southern boundary conditions of the coarser grid model are, respectively, given by

$$v + (g/h)^{1/2}\zeta = 0, \tag{6}$$

$$v - (g/h)^{1/2}\zeta = 0, \tag{7}$$

$$u - (g/h)^{\frac{1}{2}}\zeta = -2(g/h)^{\frac{1}{2}} a \sin\left(\frac{2\pi t}{T} + \varphi\right), \tag{8}$$

where a, φ and T are the amplitude, phase and period, respectively, of the tidal constituent.

3. The data sources

Bathymetric data of the present study were compiled from the British Admiralty Chart (BAC). The meteorological input data were collected from the Bangladesh Meteorological Department (BMD). The observed water level data were procured from the Bangladesh Inland Water Transport Authority (BIWTA).

4. A short note on the RKARMS(4,4) method

Basically, the embedded methods are designed to produce an estimation of the LTE to control the error with adaptive step-size (h). They are actually two methods built into one, one with order p and one with order