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Review

Analytical solution for the linear wave diffraction by a uniform vertical cylinder with an arbitrary smooth cross-section

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A B S T R A C T

An analytical method is proposed to investigate the wave diffraction of linear waves with a uniform, bottom-mounted cylinder with an arbitrary smooth cross-section. Based on the condition that the radius function of the cylinder surface can be expanded into a Fourier series, the linear diffraction theory is extended to solve the diffraction problem of linear waves in such large-scale structures. The present method is first validated using a uniform vertical cylinder with cosine-type radial perturbations. Then, the wave diffraction, wave force and wave run-up are investigated for such structures under wave attacks with different rotation angles. Finally, this method is further extended to a practical engineering application in a quasi-ellipse caisson foundation for a cross-strait bridge pylon. The results show that the method that we have developed can be effectively used for predicting the wave force and wave run-up of large-scale cylinders with arbitrary smooth cross-sections considering the wave diffraction effects.

Contents

1. Introduction

In offshore engineering, an ocean wave is a major load that threatens the safety of structures in a marine environment. For a large-scale body, quantitative understanding of the effects of diffraction due to wave-structure interactions is of primary importance to determine the wave force acting on the coastal structures when subjected to ocean wave actions. Linear diffraction theory is a common method to theoretically analyze the interaction of a linear wave with a cylinder based on the potential theory. An analytical solution to the interaction between linear waves and a bottom-fixed vertical circular was initially proposed by Havelock (1940) for the deep-water case. Later, this theory was extended by MacCamy and Fuchs (1954) to a finite water depth. The experimental study of Chakarabarti and Tam (1975) demonstrated that the linear diffraction theory was reasonably accurate to predict the wave force on a circular cylinder if $2A/h \le 0.25$ (A is the wave amplitude, and h is the water depth) and $0 \le ka_0 \le 3$ (k is the wave number and a_0 is the cylinder radius). Nevertheless, the linear diffraction theory is no longer suitable for calculating the wave action of a large body under a strong nonlinear wave. Therefore, many studies were also conducted by researchers to achieve an exact estimation of the wave force on a circular cylinder under nonlinear wave actions (Chau and Taylor, 1992; Lighthill, 1979; Malenica and Molin, 1995; Molin, 1979; Newman, 1996).

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Except for a circular cylinder, the wave-structure interaction problem of a bottom-fixed cylinder with some other specific geometric shapes of the cross-section was also addressed by researchers to obtain an analytical solution. Chen and Mei (1973) presented an exact solution of wave forces acting on an elliptical cylinder by the Mathieu function in elliptic cylindrical coordinates. By this method, the complete solution is very complex due to the requirement of calculating the infinite series of the Mathieu function. To reduce the computational work, Williams (1985) developed two alternative methods for solving the same problem.

For the numerical methods to simulate wave loading on large-scale objects with arbitrary shapes, Green's function plays an important role for the analysis of this class of problems. Recently, a detailed history and discussion of Green's function were presented by Duffy (2015). The original concept of Green's function came from classical electrostatics, and this concept enjoyed great success in the classic field of an irrotational water wave. Cauchy and Poisson first applied Green's function to solve the two-dimensional problem of the water wave surface in the nineteenth century. Later, Green's function was studied extensively during the 1940s and early 1950s, and several alternative integral representations were given, as reviewed by Wehausen and Laitone (1960). For the wave problem of the square caisson, Isaacson (1978) developed a method by assuming a distribution of vertical line wave sources over the submerged body surface. Mansour et al. (2002) presented two methods to analyze the wave diffraction of linear waves by a uniform vertical cylinder with cosine-type radial perturbations. In this study, an analytical solution based on perturbation theory was developed for small perturbation amplitudes of the circular cross-section. Nevertheless, a boundary element solution, which is similar to the solution of Isaacson (1978), was also presented in that study for the case of no restriction on the magnitude of the perturbation amplitude based on Green's theorem.

Furthermore, to overcome the defect that the wave source is not effective for calculating the hydrodynamic forces when the cylinder is oscillating, the local disturbance source was introduced by Miao and Liu (1990) and Miao et al. (1993) to solve the hydrodynamic forces acting on a single cylinder with an arbitrary cross-section vibration in still waters. In the study of Ghalayini and Williams (1989), the first-order potential on a vertical cylinder with arbitrary cross-section, expressed in terms of eigenfunction expansions, was calculated by using Green's function. In this study, the second-order wave force was also calculated by an efficient numerical technique. For the wave loading acting on the arbitrary shape, some researchers also addressed the numerical method to solve this type of problem. Methods such as the finite element method (Shankar et al., 1984), boundary element method (Au and Brebbia, 1983; Zhu and Moule, 1994). Recently, Tao et al. (2007) used the scaled boundary finite element method, which is a semi-analytical method developed in the elasto-statics and elastodynamics areas, to solve the boundary-value problem composed of short-crested waves diffracted by a vertical circular cylinder. This method was also utilized by Song et al. (2010) to analyze the water wave interaction with multiple cylinders of arbitrary shape. The analyzed results indicated that this method has a great advantage in treating the cylinders with prismatic surface. Naserizadeh et al. (2011) developed a BEM-FDM technique to solve the modified mild slope equation by using the combination of the boundary element method (BEM) and the finite difference method (FDM). The main idea of this method was to utilize BEM in the exterior domain with constant depth and FDM in the interior domain with variable depth. The refraction and diffraction problem of waves from submerged bottom mounted obstacles was analyzed and compared well with experimental measurements. Focused on the wave-power farm, McNatt et al. (2013) developed a new method for computing the cylindrical wave-field coefficients for an arbitrary geometry. In this study, the Fourier transform and the orthogonality property of the depth dependence was employed, and the circular-cylindrical section of the wave field was computed with the boundary-element-method solver.

With a current trend of more cross-strait bridges being built in deeper waters (Feng, 2013), the structural safety of coastal bridges in a marine environments, especially for the long-span navigation bridges with a large-size foundations, becomes more and more important when the bridge is subjected to wave loading and the combined action of waves and other types of natural hazards. For most coastal bridges, some specific geometrical shapes rather than a circular cylinder (such as a quasi-ellipse caisson) have been selected for the foundation of the pylons. Understanding the wave-structure interaction mechanism and establishing an accurate method to predict the wave loading acting on such types of foundations are important issues for researchers and engineers. However, most of the analytical studies on the wave-structure interaction on a bottom-fixed cylinder mainly focus on a cross-section with circular and elliptical shapes.

In this article, the linear diffraction theory is extended to solve the wave force and wave run-up on a bottom-fixed uniform cylinder with an arbitrary smooth cross-section in which the radius function can be expanded into a Fourier series. The main contents of this study are organized as follows. First, the definition of the physical problem and the mathematical derivation of the analytical solution of the scattered-wave potential is presented in Section 2. Then, the proposed method is validated by the vertical uniform cylinder with a noncircular section, and the comparative results are introduced in Section 3.1. Focused on the circular cylinder with cosine-type perturbations, the wave diffraction, wave force and wave run-up are investigated and discussed in Section 3.2 considering the effects of rotation angle, shape perturbation and wave number. This method is further extended to a practical engineering application with a quasi-ellipse caisson foundation of a cross-strait bridge pylon in Section 3.3. The main findings of the present work are summarized in the final section.

2. Mathematical formulation

Fig. 1 shows the schematic diagram of the wave diffraction around a uniform surface-piercing cylinder, which is assumed to be rigid and mounted at the bottom of the seedbed. In the analysis, the origin of the coordinate system is set inside the cross section at the still water level (SWL). In polar coordinates, r and θ are defined in the horizontal plane, and the z-axis is perpendicular to the SWL and positive upward.

Under the action of gravity, the water wave is assumed to be an ideal fluid with incompressible, inviscid and irrotational characteristics. Based on these conditions, the total velocity potential of the fluid, $\Phi(r, \theta, z, t)$, can be written in a complex form as

$$\Phi(r, \theta, z, t) = \left[\phi_I(r, \theta, z) + \phi_D(r, \theta, z)\right]e^{-i\omega t} \tag{1}$$

in which $\phi_I(r, \theta, z)$ and $\phi_D(r, \theta, z)$ are the spatial velocity potential of the incident wave and the scattered wave, respectively; ω is the circular frequency of incident wave. In polar coordinates, the velocity potential of the incident wave can be expressed as

$$\phi_I(r, \theta, z) = -i\frac{Ag}{\omega}\frac{\cosh k(z+h)}{\cosh kh}\sum_{m=0}^{\infty}\varepsilon_m i^m J_m(kr)\cos m\theta \tag{2}$$

in which g is the gravity acceleration; A is the wave amplitude; k is the wave number, which is related to the wave frequency through the dispersion equation $\omega^2 = gk\tanh(kh)$; h is the water depth; and