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Rigid-flexible-cavity coupling trajectory and uncertainty trajectory analysis of supercavitating projectiles

Ling Zhou*, Yanhui Li, Zhipeng Xue, Jingzhuang Han, Nan Zhang

Department of UAV, Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences, Changchun City 130033, PR China

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ABSTRACT

Structures of supercavitating projectiles operating at high underwater velocity are subjected to large deformations generated by high forces acting on the projectile. Moreover, it is widely that probabilistic and non-probabilistic uncertain information are coexisting. Therefore, flexible and uncertainty trajectory analysis of supercavitating projectiles are required. Formulae of flexible body motion and rigid-flexible coupling dynamic differential equations are introduced first. Then rigid-flexible coupling equations are decoupled. Flexible deformations are solved by modal superposition method; detailed rigid trajectory equations in vertical plane, supercavity equation and force formulae are presented, and calculation flowchart of rigid-flexible-cavity coupling trajectory is given. Third, by chaos method, the uncertainty rigid-flexible-cavity coupling trajectory simulation of supercavitating projectiles with uncertain launch parameters is performed, and parameters are described by random variables and non-probabilistic interval variables. Finally, The correctness of rigid-flexible-cavity coupling trajectory algorithm is validated by the experimental data provided by relevant literatures. To investigate the effect of flexible deformation of projectile on coupling trajectory, the variation of the resultant coupling trajectory was investigated by varying two important flexibility parameters – slenderness ratio and Young's modulus. The results of rigid-flexible-cavity coupling trajectory are compared with that of rigid-cavity trajectory through an engineering example. Trajectory curves sets and distribution of impact points are presented through uncertainty rigid-flexible-cavity coupling trajectory simulation.

1. Introduction

The local pressure of a fluid decreases as the bodies move at sufficient high speed through water, and then low-density gas generates. The gas forms a cavity, which envelops the moving body according to a phenomenon commonly known as “supercavity” (Ashley, 2001). Supercavitating bodies can reach high speeds in water, such as the speed of a supercavitating projectile used in experiments is approximately 1500 m/s (Harkins, 2001). To meet the requirements of fluid dynamics enveloped by a supercavity, supercavitating projectiles are usually designed as slender bodies. However, the high slenderness ratio combined with the large axial force caused by the cavitator drag and the sliding force, which are proportional to the square of velocity, may easily cause large deformations of structure in the supercavitating projectiles. Therefore, when performing trajectory analysis of supercavitating projectiles, large deformations of structure need to be considered.

At present, researches on the supercavitating vehicles mainly focus on hydrodynamics, control, and buckling problems (Semenenko, 2004; Zhang et al., 2015; Ruzzene, 2004) and only a few trajectory analysis, especially flexible trajectory calculation and uncertainty trajectory analysis of supercavitating vehicles, have been carried out. Kulkarni and Pratap (2000) studied the rigid dynamics of a supercavitating projectile. Rand et al. (1997) investigated the in-flight dynamics of a simplified model of a supercavitating body, Ruzzene et al. (2008) and evaluated the optimal rigid-cavity coupling trajectories for supercavitating vehicles. Choi et al. (2004) used Modal-Based Elements to perform dynamic analysis of flexible supercavitating vehicles. The dynamic behaviour of supercavitating underwater vehicles and the vibration in supercavitating underwater vehicles were investigated and controlled by Ruzzene and Soranna (2004). Cavity formula and sliding force formula reported in Choi et al. (2004) and Ruzzene and Soranna (2004), do not take into account the unsteady supercavity characteristics, and the value of the constant in sliding force formula should be obtained by experiments.

Nonetheless, the parameters are commonly uncertain in engineering, so deterministic trajectory analyses are not adequate. Commonly there are uncertain factors in initial launch process, which will effect on the resulting trajectory. For example, the

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* Corresponding author.
E-mail address: hszl007@163.com (L. Zhou).

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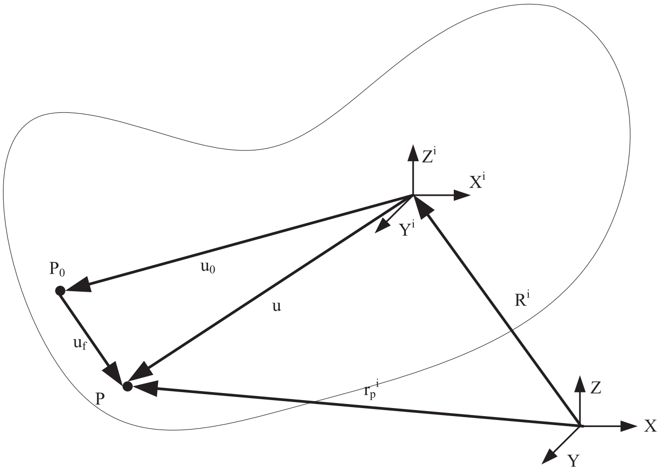

Fig. 1. Flexible body dynamic.

initial velocity of the launch, initial angular velocity and pitch angle are not determined in supercavitating projectile's launch process. Due to disturbances generated during launch by manufacturing errors of supercavitating projectiles and by launch equipment, flow disturbances, and launch equipment pressure disturbances the above-mentioned initial launch parameters are uncertain. Jiang performed trajectory stochastic characteristics analysis of supercavitating projectile with stochastic parameters (Yunhua et al., 2011), but the flexible characteristic of vehicle was not considered and the distributions of random variables were infinite. However, truncated random variables should be used instead of continuous random variables for practical engineering applications because most of random variables are bounded (Xiao et al., 2014).

Besides truncated random variables, there are non-probabilistic interval variables, and it is common that the above two uncertainties are coexisting because there are many uncertain variables with various data samples and various characteristics in engineering. For example, uncertain variables can be described as random variables when data samples are adequate and an accurate probability distribution can be obtained. However, an accurate probability distribution cannot be obtained with a too small sample of data. Small changes in the probability distribution of uncertain variables can generate large differences in the probabilistic analysis results (Elishakoff, 1995). Therefore, uncertainty variables cannot be treated as random variables when their sample data are too few to obtain accurate probabilistic analysis results. However, the boundaries of uncertain variables may be easily obtained from engineers and test data. Therefore, the use of a non-probabilistic interval variable to describe uncertain variables is appropriate (Ben-Haim, 1994).

According to the above analysis, flexible characteristics of supercavitating projectile, unsteady supercavity, and uncertainties of the launch parameters must be considered. Therefore, it is necessary to perform uncertainty rigid-flexible-cavity coupling trajectory analysis of supercavitating projectiles.

In this paper, rigid-flexible-cavity coupling trajectory and uncertainty trajectory analysis of supercavitating projectiles are presented. The unsteady supercavity characteristics, directional effect of cavitator and floating deformation of cavity tail will be considered and a more accurate calculation formula of sliding lift force will be used. The paper is organized as follows. Flexible body motion analysis and rigid-flexible coupling equations are presented in Section 2. Rigid-flexible coupling equations are decoupled, and flexible deformations are solved by modal superposition method in Section 3. Detailed rigid trajectory equations in vertical plane, supercavity equations, force formulae and solving flowcharts are presented in Section 4. Uncertainty variables are generated by chaotic variables and simulation flowcharts of uncertain trajectory are presented in Section 5. In Section 6, numerical examples of supercavitating projectiles and main results are presented. Finally, conclusions are drawn in Section 7.

2. Flexible body motion analysis

In Fig. 1, *XYZ* is a global coordinate system, and *XiYiZi* is a body coordinate system, so the global coordinate of arbitrary point P on the body is expressed as

$$r_p^i = R^i + A^i u \tag{1}$$

where, R^i is position vector of body in global coordinate system, A^i is rotation matrix from local coordinate system to global coordinate system, u is position vector of point P in local coordinate system. Flexible deformations were considered and u could be expressed as

$$u = u_0 + u_f \tag{2}$$

where, u_0 and u_f are undeformed position vectors and deformation displacement vectors in local coordinate system, respectively. Then, after separating the body by FEM, the deformations of the nodes were

$$u_f = Nq_f \tag{3}$$

where, q_f are deformations of nodes, N is shape function. The global position of P could be expressed by both reference coordinate system and floating coordinate system as follows

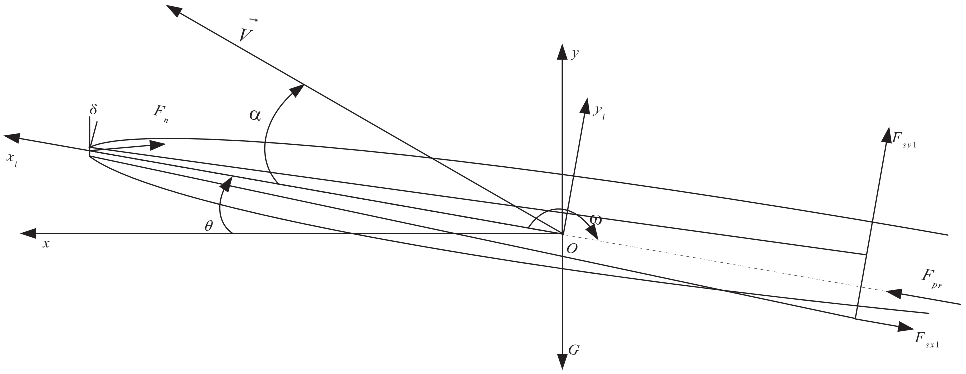

Fig. 2. Forces acting on a supercavitating projectile.