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Wavelet based spectral algorithm for nonlinear dynamical systems arising in ship dynamics

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ABSTRACT

In this paper, we have applied an efficient shifted second kind Chebyshev wavelet method (S2KCWM) to vibrating dynamical models arising in mechanical systems such as vibration of circular membrane, damped spring mass system and ship oscillatory motions. To the best of our knowledge, until now there is no rigorous wavelet based solution has been reported for the vibrating dynamical models. The power of the manageable method is confirmed. The wavelet solutions are compared with numerical simulations by MATLAB. Good agreement between the solutions is presented in this paper. Some numerical examples are given to demonstrate the validity and applicability of the proposed method. Moreover the use of Chebyshev wavelets is found to be simple, efficient, flexible, convenient, less computation costs and computationally attractive.

1. Introduction

In recent years, membrane dynamics model is a classical problem in mechanical vibrations. Among the several types of membranes, circular membranes are the most widely studied due to their numerous applications in engineering (Agarwal and D.O'Regan, 2003; Shin, 1995; Javidinejad, 2013; Ji- Ping and Xin-LI, 2006; Siedlecka et al., 2012; Civalek and Gürses, 2009; Hsu, 2007; Chapra and Canale, 2002). From the study of musical notes of percussion instruments, circular membranes have been used to design diaphragms for condenser microphones, model the dynamics of the human ear (Alsahlani and Mukherjee, 2013), understand the vibration characteristics of membrane mirrors and gossamer structures (Alsahlani and Mukherjee, 2013), measure surface tension (Alsahlani and Mukherjee, 2013; Mgharbel et al., 2009), and design ink-jet printers (Alsahlani and Mukherjee, 2013). The similarity between the differential equations of membranes and waveguides motivated the study of circular membranes with constraints in the 1970s and 1980s (Krenk and Schmidt, 1981). Sen et al., (2006) established the interpolation for nonlinear boundary value problems (BVPs) in circular membrane with known upper and lower solutions. Recently, Alsahlani and Ranjan Mukherjee (Alsahlani and Mukherjee, 2013) had introduced the dynamics of a circular membrane with an eccentric circular areal constraint.

Considerable attention has been directed toward the chaos, chaotic systems and solutions of nonlinear oscillator differential equations since they play crucial role in natural and physical simulations. Surveying the literature shows that a variety of solution methods have been developed so far to solve the duffing oscillator equation (Cvetićanin, 2009; Trueba et al., 2000; Huang and Zhu, 2012; Kim and Park, 2015; Nourazar and Mirzabeigy, 2013; Cveticanin, 2011; Joseph and Minh-Nghi, 2005; Sharma et al., 2012; Zhu, 2014; Kaur et al., 2014). Some researchers in their studies consider damping into the duffing oscillator. When the duffing oscillator involves damping, the amplitude of oscillation reduces over time and we have a non-conservative system. Most analytical methods are unable to handle non conservative oscillators. Our aim in the present study is to obtain the solution of the duffing oscillator free response considering different damping effects and with different initial conditions by the second kind Chebyshev wavelet method and comparing the results with the results of a numerical solution using the MATLAB.

Roll motion is a major concern of ship and offshore operators. The major technical difficulties related to the roll motion of a floating body are the nonlinear effects of roll damping (Huang and Zhu, 2012; Kim and Park, 2015; Bulian, 2004). A highly nonlinear characteristic is strongly involved in the ship roll motion models. It is necessary that the dynamic stability of ships in realistic sea is dependent on its rolling motion and therefore the investigation of ship's roll dynamics is most crucial unlike other degrees of freedom of ship motion. For this purpose, it is generally required to investigate ship's roll damping for accurate and efficient prediction of its response to various loading environments and development of control strategies: this is essential for the design of ship-shaped structures. However, the determining of the roll damping is difficult because of its strong nonlinearity.

Wavelet analysis has found their way into many different fields in

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science, engineering and medicine. It possesses many useful properties, such as Compact support, orthogonality, dyadic, orthonormality and multi-resolution analysis (MRA). Recently, wavelets have been applied extensively for signal processing in communications and physics research, and have proved to be a wonderful mathematical tool. After discretizing the differential equations in a conventional way like the finite difference approximation, wavelets can be used for algebraic manipulations in the system of equations obtained which lead to better condition number of the resulting system (Adibi and Assari, 2010; Li, 2011; Sohrabi, 2011; Mason and David, 2002; Hariharan et al., 2009; Hariharan and Kannan, 2009, 2010; Ghasemi and Kajani, 2011; Abd-Elhameed et al., 2013; Gh. et al., 2011; Barzkar et al., 2012; Hariharan and Kannan, 2013; Hariharan, 2014a; Rajaraman and Hariharan, 2015; Hariharan and Rajaraman, 2013; Mahalakshmi et al., 2013; Pirabaharan et al., 2015).

There is a growing interest in using various wavelets to study problems, of greater computational complexity. Among the wavelet transform families the Haar, Legendre and Chebyshev wavelets deserve much attention. The basic idea of Chebyshev wavelet method (CWM) is to convert the differential equations in to a system of algebraic equations by the operational matrices of integral or derivative. The main goal is to show how wavelets and multi-resolution analysis can be applied for improving the method in terms of easy implementability and achieving the rapidity of its convergence. Wavelets, as very well-localized functions, are considerably useful for solving differential equations and provide accurate solutions. Also, the wavelet technique allows the creation of very fast algorithms when compared with the algorithms ordinarily used (Hariharan et al., 2009; Hariharan and Kannan, 2009, 2010). Recently, Hariharan and Kannan (Hariharan and Kannan, 2014) reviewed the wavelet transforms methods for solving a few reaction-diffusion equations arising in science and engineering. In this paper, the shifted second kind Chebyshev wavelet method (S2KCWM) is applied to vibrating circular membrane model arising in mechanical vibrations. The method consists of reducing the differential equations to a set of algebraic equations by first expanding the candidate function as Chebyshev wavelets with unknown coefficients (Adibi and Assari, 2010; Li, 2011; Sohrabi, 2011; Mason and David, 2002; Ghasemi and Kajani, 2011; Abd-Elhameed et al., 2013; Abd-Elhameed et al., 2013; Gh. et al., 2011; Barzkar et al., 2012; Babolian and Fattahzadeh, 2007; Zhu and Fan, 2012; Doha et al., 2013; Doha et al., 2013; Hariharan, 2013). Recently, Heydari et al.Heydari et al., 2015 have established the wavelet Galerkin method for solving stochastic heat equation.

This paper is organized as follows. In Section 2, some properties of shifted second kind Chebyshev polynomials are presented. Some properties of shifted second kind Chebyshev wavelets are presented in Section 3. Some numerical examples are given in Section 4. Concluding remarks are given in Section 5.

2. Some main properties of the second kind Chebyshev polynomials and their shifted form (Hariharan, 2014b)

It is well known that the second kind Chebyshev polynomials are defined on [−1,1] by

$$U_n(x) = \frac{\sin (n+1)\theta}{\sin \theta}, \quad x = \cos \theta. \tag{1}$$

These polynomials are orthogonal with respect to the weight function $w(x) = (1-x^2)^{\frac{1}{2}}$ on the interval [−1, 1]

$$\int_{-1}^{1} \sqrt{1-x^2}\, U_m(x) U_n(x) dx = \begin{cases} 0, & m \neq n \\ \dfrac{\pi}{2} & m = n \end{cases} \tag{2}$$

The following properties of second kind Chebyshev polynomials are of fundamental importance in the sequel. These basis polynomials are eigen functions of the following singular Sturm-Liouville equation.

$$(1-x^2)U_k''(x) - 3xU_k'(x) + k(k+1)U_k(x) = 0, \tag{3}$$

$$U_{k+1}(x) = 2xU_k(x) - U_{k-1}(x), \qquad k = 1, 2, 3, \ldots \tag{4}$$

Starting from $U_0(x)=1$ and $U_1(x)=2x$, or from Rodrigues formula

$$U_n(x) = \frac{(-2)^n (n+1)!}{(2n+1)!\sqrt{(1-x^2)}} \frac{d^n}{dx^n}[(1-x^2)^{n+\frac{1}{2}}]. \tag{5}$$

Theorem 2.1: The first derivative of second kind Chebyshev polynomials is of the form.

$$\frac{d}{dx}U_n(x) = 2 \sum_{\substack{k=0 \\ (k+n) odd}}^{n-1} (k+1)U_k(x). \tag{6}$$

Definition (Hariharan, 2014b):

The shifted second kind Chebyshev polynomials are defined on [0,1] by$U_n^*(x) = U_n(2x-1)$. All mentioned properties of the second kind Chebyshev polynomials can be easily transformed for their corresponding shifted form. It should be noted that the shifted second kind Chebyshev polynomials are orthogonal with respect to the weight function $w^*(x) = \sqrt{x-x^2}$on the interval$[0, 1]$,

i.e.,

$$\int_0^1 \sqrt{x-x^2}\, U_n^*(x)U_m^*(x)dx = \begin{cases} 0, & m \neq n \\ \dfrac{\pi}{8}, & m = n. \end{cases} \tag{7}$$

Corollary (Hariharan, 2014b): The derivative of the shifted second kind Chebyshev polynomial can be expressed as

$$\frac{dU_n^*(x)}{dx} = 4 \sum_{\substack{k=0 \\ (k+n)\ odd}} (K+1)U_n^*(x). \tag{8}$$

3. The second kind Chebyshev wavelets and their properties (Hariharan, 2014b)

Second kind Chebyshev wavelets are denoted by$\psi_{n,m}(t) = \psi(k, n, m, t)$, where k, n are positive integers and m is the order of second kind Chebyshev polynomials.

Here t is the normalized time. They are defined on the interval [0,1] by

$$\psi_{n,m}(t) = \begin{cases} \dfrac{2^{\frac{k+3}{2}}}{\sqrt{\pi}} U_m^*(2^k t - n), & t \in \left[\dfrac{n}{2^k}, \dfrac{n+1}{2^k}\right], \\ 0 & otherwise \end{cases} \tag{9}$$

m=0,1,..... M, n=0,1,...2k−1.

A function f (t) defined over [0,1] may be expanded in terms second kind Chebyshev wavelets as

$$f(t) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} c_{nm}\psi_{nm}(t)., \tag{10}$$

Where

$$c_{nm} = (f(t), \psi_{nm}(t))_w = \int_0^1 \sqrt{t-t^2} f(t)\psi_{nm}(t)dt. \tag{11}$$

If the infinite series is truncated, then it can be written as

$$f(t) \approx \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} c_{nm}\psi_{nm}(t). = C^T\psi(t). \tag{12}$$

where C and ψ (t) are 2k(M+1) x 1 column vectors defined by