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Approximation of the joint probability density of wave steepness and height with a bivariate gamma distribution

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ABSTRACT

A bivariate gamma distribution is fitted to an empirical joint probability density of wave steepness and height for deep water waves. A transformation of this distribution is also fitted to waves from a wave tank experiment in order to observe how the skewness of the bivariate gamma distribution changes. The quality of the obtained fits is compared with fits of a Gumbel copula for the same data.

1. Introduction

The effect of waves on marine structures is mostly dominated by their height, which is probably the most studied and understood wave parameter as many probabilistic models are available to describe its short term and long term behavior. For floating structures their dynamic behavior is also important and thus there are available models of joint distribution of wave height and period.

The steepness of a sea state relates its significant wave height with the characteristic period and this has been determined as important to evaluate the loading on floating structures, (Bitner-Gregersen et al., 1998) which tends to be higher in steep sea states and it was even found that there is a higher probability of ship losses in areas with steeper sea states (Guedes Soares et al., 2001).

However for the case of impact loads even the steepness of individual waves is of importance, studied in an extensive experimental program related with damages on the bows of FPSOs (Guedes Soares et al., 2004a, b). It was observed that the local wave steepness, which is related to the free surface vertical velocity, was the governing parameter as it was found in experiments that wave slams occurred only for high values of local steepness.

Despite the importance of knowing the individual water wave steepness, probabilistic models of this wave parameter in a sea state are not yet successfully described with any theoretical distribution. Overall, there is not much literature devoted to individual wave steepness probabilities and there is no model of probability distribution of individual wave steepness for a sea state.

A complete set of steepness and asymmetry parameters for individual waves was proposed by Guedes Soares et al. (2004a). For the largest waves, the correlation of these parameters with different individual wave statistics was described, too. Although, most often the classical (or standard) wave steepness is investigated, sometimes it is necessary to use different definitions of steepness characteristics in a study, just like in Guedes Soares et al. (2004b).

Myrhaug and Kjeldsen (1984) presented parametric models for the joint probability density of wave steepness and asymmetry for deep water waves at sea of the Norwegian continental shelf. Their study was initiated after the loss of 26 vessels and 72 lives in the period 1970–79. In 13 cases survivors confirmed that the vessels capsized due to large breaking waves. In the rest of the cases the most probable cause of the accident was determined to be the same. These events triggered investigation on steepness and asymmetry of deep water waves in Norway. The models developed by Myrhaug and Kjeldsen (1984) were elaborated for total wave steepness and crest front steepness, and they are valid for steep asymmetric waves and breaking waves in deep water. The models presented in Myrhaug and Kjeldsen (1984), just like most of the existing steepness models, were prepared for set of waves pooled although coming from different sea states recorded in extreme conditions. However the work presented in the next sections is devoted to the individual wave steepness of a singular sea state.

Another work on steepness and asymmetry by Myrhaug and Kjeldsen (1986) treats high as well as extreme waves, which are more rare events. The authors showed correlation between different wave statistics connected to steepness and investigated the profile of an extreme wave.

Stansell et al. (2003) presented a model for extreme storm conditions. It predicts the probabilities of extreme wave steepness given dimensionless wave height. The model was developed for northern North Sea wave conditions. One more joint model based on different records collected in storms was prepared by Linfoot et al.

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(2000). In that publication the bivariate probability density of wave height and steepness was approximated with a bivariate Weibull model. It is one of very few works that used theoretical bivariate probability distribution to describe steepness and another characteristic of ocean waves.

Hamadeh (1989) derived a theoretical expression for the steepness distribution based on the joint distribution of wave heights and periods of Cavanié et al. (1976). Extensive comparisons with oceanic data showed that this model does not give successful fits to empirical steepness distributions.

Many authors were trying to fit the univariate steepness probability density with different probabilistic distributions. Askar et al. (1995) showed that the best fit of the empirical steepness distribution is obtained with the lognormal distribution, while Overvik and Houmb (1977) suggested the Rayleigh distribution. Myrhaug and Rue (1993) used the two-parameter Weibull distribution.

Tayfun (2006) verified few of the above mentioned models against different sets of storm data and concluded that the best approximation for wave steepness was the lognormal distribution as long as waves were long-crested and were part of a narrow band spectrum.

Wave steepness defined differently was studied by Bittner-Gregersen et al. (1995). On the basis of long series of extreme storm conditions mathematically simulated and generated in an offshore basin, a comparison between 2nd order and 3rd order nonlinear models was presented.

For many applications more interesting than the model of univariate steepness distribution, are models of joint distribution of steepness and another wave characteristic. Due to the fact, that wave statistics like period, crest, height, length and steepness are not independent from each other the interest in joint distributions is growing as do the requirements for more precise statistical description of ocean waves. Additionally probability distributions in a form of continuous functions are more comfortable to use and infer from than empirical discrete distributions. For example from a continuous distribution of two wave characteristics one can obtain distribution of different wave characteristics just through change of variables. Also a distribution for one sea state gives more precise information about what one can expect in given sea conditions than a distribution prepared on the basis of a set of waves collected in different sea states.

The main subject of this work is modeling of joint distribution of steepness and height for a given sea state. One approach would be to start from the bivariate distributions of wave height and period as proposed by Longuet-Higgins (1983). This was attempted by Antão and Guedes Soares (2015), but difficulties were experienced resulting from the position of the peak period.

This paper searches for a good description of random wave steepness and height with a joint model, bivariate gamma distributions are fitted to the measured data. Next the bivariate gamma distribution is transformed to yield the Rayleigh distribution as the marginal for wave heights. The fits obtained with bivariate distributions are then compared to fits which gave Gumbel copula in Antão and Guedes Soares (2014). Before the comparison, the bivariate gamma probability density is transformed to obtain an asymmetric function.

In this work tank data are used. They have greater length than the available series collected at sea. This together with fact that the series were generated on the basis of one peak spectrum should ensure better smoothness of empirical histograms without interfering with their real shape. Time series generated in a tank have also small sampling interval and more precise information on number and direction of component wave systems and current, and that is also helpful especially when comes to the last stage of construction of the model that is to parameterization with sea state characteristics.

An important problem in steepness modeling is that steepness of every wave constantly changes as this wave travels and the standard (or total, called also classical) wave steepness defined as: S = H/L is not enough to describe an ocean wave. Nevertheless sufficiently long records could compensate for the lack of information on changes of the steepness value of a wave during its evolution. Simply, longer records contain more waves from the given population and these waves could be registered in sufficient number of life stages to describe the population.

Table 1
Description of sea states from DHI offshore basin.

Test Number	Hs [m]	Tp [s]	No waves in series
1	3,6	7	821
2	3,6	10	624
3	3,6	14	469
4	3,6	20	328
5	4,6	7	801
6	4,6	14	461
7	4,6	20	324
8	2,3	7	819
9	2,3	14	469
10	2,3	20	334
11	3,6	7	745
12	3,6	14	430
13	4,6	14	425
14	2,3	14	447
105	5	10	628
106	5	10	588
107	5	12	535
108	10	12	512

2. Description of the experimental data

One of the reasons of problems with the investigation of the distribution of individual wave steepness is the quality of measurements of water elevation in the open sea. Every existing type of equipment to measure water elevation has its limitations, and during recording the real surface wave profile is usually distorted. Another reason of the problems can be too big sampling interval and the small length of real sea surface records. That justifies the use of tank data in this work. They have smaller sampling interval and greater length than series collected at sea and it usually results in better smoothness of histograms.

The data were generated and recorded in an offshore basin of the Danish Hydraulic Institute in Hørsholm (DHI). They are time series of instantaneous water elevation from which gravity waves of deep water were obtained. All the 110 series were recorded by the same wave gauge placed in the center of the wave basin. The data was scaled in DHI with factor 1:75 using the Froude law. The time interval of sampling after scaling was $\Delta t=0.2165$ s. In the majority of files there are N= 24,001 ordinates per record, what gives 86 min long time series of full scale.

The 18 records used here contain sea states with 2D waves of one wave system and without current in different combinations of significant wave height and spectral peak period values. More precise description of sea states is given in Table 1 which shows, in addition to the significant wave height and spectral peak period, also the direction of waves. There are designed values in the table. The respective full scale values calculated directly from recorded waves can differ a little from the designed ones. The differences are of order 1–5 cm in case of Hs and are smaller than 0,4 s in case of T_p.

The first ten sea states are different combinations of three significant height values with four spectral peak period values. The last group of 4 sea states is of higher values of significant wave height. Sea states 11–14 are of basic parameter values chosen among the previous ten ones.

An important thing is that all the spectra obtained from the records in Table 1 are of one peak, moreover it was verified that the global minimum of autocorrelation function of each series is at the same time