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Modeling the spatial evolutions of nonlinear unidirectional surface gravity waves with fully nonlinear numerical method

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ABSTRACT

Statistical properties of mechanically generated unidirectional nonlinear wave series are simulated with the fully nonlinear Chalikov-Sheinin model, which is based on a non-stationary conformal surface-following coordinate transformation that reduces the principal equations of potential waves into two simple evolutionary equations for the surface elevation and the velocity potential on the surface. Meanwhile, the numerical simulations performed by the temporal version of modified nonlinear Schrödinger equation (MNLS) are also compared with the laboratory experiments in this study. As a result, except for the wave height distribution, Chalikov-Sheinin or simply called CS model performs a little better in simulating the observed statistics than MNLS equation, particularly in the aspects of fourth-order normalized cumulants and the intermediate probability of wave crest and trough exceedance distributions in the presence of strong nonlinearity.

1. Introduction

To determine the statistical properties of wave amplitudes is a very important task in the study of surface gravity waves for the reason that the probability of occurrence of large amplitude waves is critical in a variety of engineering applications. For example, the statistical distribution of crest elevations must be established with care for an input to the wave load calculations, and the distribution of wave troughs is of significance for the determination of the maximum trough depth in the design of offshore platforms (Toffoli et al., 2008).

Although the traditional linear theory predicts Gaussian statistics for the wave surface, it is commonly reported that the surface elevation, for instance in deepwater condition, is slightly non-Gaussian. In the frame of weakly nonlinear narrow-band theory, it is often assumed that the wave field can be decomposed into two parts: a superposition of free waves that satisfy the dispersion relation, and the nonlinear bound waves that do not satisfy the dispersion relation but are phase-locked to the free waves. On one hand, the free waves are assumed to yield Gaussian statistics and the nonlinear bound waves produce small corrections to Gaussian statistics, known as Tayfun distribution (Tayfun, 1980). On the other hand, the nonlinear interactions among free waves, called Benjamin-Feir instability (Benjamin and Feir, 1967) or modulational instability (Zakharov, 1968), can give rise to a significant deviation from Gaussian statistics and lead to increased occurrence of freak waves in condition of narrow initial spectrum (Shemer et al., 2010a, 2010b) and insignificant directional spreading. Working together, the departure from normal distribution can be further enlarged, contributing to some new relations among those fourth-order normalized cumulants (Zhang et al., 2015a, 2015b). Hence, to capture the more subtle features of surface waves, the fully nonlinear numerical model has to be employed even though it will require considerable computational resource.

Research on the numerical simulations of surface waves has been made for a long time, and the most general approach to simulate a motion with a free surface is based on a marker and cell (MAC) method (Harlow and Welch, 1965), which can trace a variable surface within a fixed grid with different orders of accuracy. This method is recently restricted to simulations over relatively short-term periods and its accuracy can be increased significantly if high resolution is possible. The advantage of this method is that it can be used to simulate 3D rotational motion of viscous fluids even for non-single value interface (Chalikov, 2005).

Although the fluid in nature is viscous and compressible, even with a rotational motion sometimes, it is fortunate that most observed properties of surface waves are reproduced well by means of the potential theory. Ignoring the influence of viscosity and assuming an incompressible condition, the primary advantage of

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the potential approximation is that the system of Euler equation is reduced to Bernoulli's equation, and the conservation of mass equation can be stated simply as Laplace equation. However, the solution to the flow problem of surface wave motion is still very complicated due to the nonlinear kinematic and dynamic boundary conditions on the free surface, the location of which is unknown at any given moment.

Another group of numerical methods are proposed on the basis of traditional perturbation expansions, and normally combined with Fourier transform method. In principle, these methods are able to include arbitrary high order interaction (Dommermuth and Yue, 1987; West et al., 1987). However, the number of required Fourier modes in this scheme multiplies with the increasing steepness. Indeed, these methods become inapplicable when waves approach overturning.

Recently, on the foundation of conformal mapping of a finite depth water domain, Chalikov and Sheinin (1996, 1998) developed a numerical scheme for direct hydrodynamic modeling of 2D nonlinear gravity and gravity-capillary periodic waves. For the stationary problem, this mapping represents the classical complex variable method originally developed by Stokes. For the nonstationary problem, the CS numerical approach allows rewriting the principal equations of potential flow with a free surface in a surface-following coordinate system. The velocity potential in the entire domain can be represented by its Fourier expansion coefficients on the free surface and the hydrodynamic system, without any simplification, is expressed by two relatively simple evolutionary equations that can be solved with a straightforward numerical method.

Moreover, the capability of this approach is that the numerical scheme permits simulating the propagation of Stokes wave with an amplitude at 98% of the maximum for hundreds of periods without noticeable distortions (Chalikov, 2005). Formally, conformal mapping exists up to the moment when an overturning volume of water touches the surface. In such kind of evolutionary process, the required number of Fourier modes has to be quickly increased. Without taking special measures such as smoothing, the numerical simulation will terminate much earlier due to strong crest instability (Longuet-Higgins, 1996), manifested by splitting of the falling volume into two phases, which is obviously non-potential.

In the present study, CS model was used to simulate unidirectional surface waves, mechanically generated in the wave basin of Marintek. Besides, considering that the modified nonlinear Schrödinger equation (MNLS) is particularly suitable to investigate wave statistics, which requires the calculation of many realizations of a random sea state (Socquet-Juglard et al., 2005; Gramstad and Trulsen, 2007), the temporal version of MNLS equation was also adopted in the numerical simulation. Thus, a systematic analysis could be made on the comparison of observed statistics and simulated results given by different nonlinear numerical models.

This paper is organized as follows: Section 2 gives a concise review on the numerical models and statistical theory used in this paper; Section 3 briefly describes the laboratory facilities of Marintek, associated experimental setup and numerical comparison; Section 4 is devoted to the detailed comparative analysis on the statistical properties of the observed and simulated long-crested wave fields; some important conclusions are summarized in the Section 5.

2. Theory

2.1. Chalikov-Sheinin model

Considering the 2D periodic deepwater wave whose dynamic is described by the principal potential equations, due to the periodicity condition, the conformal mapping can be represented by the Fourier series:

$$x = \xi + x_0(\tau) + \sum_{-M\le k\le M, k\ne 0} \eta_{-k}(\tau)\frac{\cosh\ k(\zeta+H)}{\sinh\ kH}\vartheta_k(\xi), \tag{1}$$

$$z = \zeta + \eta_0(\tau) + \sum_{-M\le k\le M, k\ne 0} \eta_{k}(\tau)\frac{\sinh k(\zeta+H)}{\sinh\ kH}\vartheta_k(\xi), \tag{2}$$

where x and z are the Cartesian coordinates. ξ and ζ are the conformal surface-following coordinates, in which H means water depth depending on time τ and η_k are coefficients of Fourier expansion of $\eta\,(\xi,\tau)$ with respect to the new horizontal coordinate ξ.

$$\eta(\xi,\tau) = h\big(x(\xi,\zeta=0,\tau),\ t=\tau\big) = \sum_{-M\le k\le M}\eta_k(\tau)\ \ \vartheta_k(\xi), \tag{3}$$

where ϑ_k denotes the function

$$\vartheta_k(\xi) = \begin{cases}\cos\ k\xi, & k\ge 0\\ \sin\ k\xi, & k<0\end{cases}, \tag{4}$$

and M is the truncation number.

Apparently, the representation of Fourier transform with definition (4) is nontraditional but actually more convenient for calculations with real numbers, as $(\vartheta_k)_\xi = k\vartheta_{-k}$ and $\sum(A_k\vartheta_k)_\xi = -\sum kA_{-k}\vartheta_k$.

Note that the definitions of coordinates ξ and ζ are expressed on the Fourier coefficients of surface elevation. It then follows from (Eqs. (1) and 2) that time derivatives x_τ and z_τ are connected by Cauchy-Riemann relations.

In the new (ξ,ζ) coordinate, the Laplace equation still retains its form due to conformity, while the kinematic and dynamic boundary equations have been changed (Chalikov and Sheinin, 1998, 2005):

$$\Phi_{\xi\xi} + \Phi_{\zeta\zeta} = 0, \tag{5}$$

$$z_\tau = -x_\xi\zeta_t - z_\xi\xi_t, \tag{6}$$

$$\varphi_\tau = -\xi_t\varphi_\xi - \frac{1}{2}J^{-1}\left(\varphi_\xi^2 - \Phi_\zeta^2\right) - z, \tag{7}$$

where (Eqs. (6) and 7) are written for the surface ζ=0 (so that z=η, as represented by Eq. (3)), and J is the Jacobian of the transformation

$$J = x_\xi^2 + z_\xi^2 = x_\zeta^2 + z_\zeta^2. \tag{8}$$

In addition,

$$\zeta_t = -\left(J^{-1}\Phi_\zeta\right)_{\zeta=0}, \tag{9}$$

and $\varphi = \Phi(\zeta=0)$, denotes the velocity potential on the surface $z=\eta$. Actually, ξ_t is a generalization of the Hilbert transform of ζ_t, which for $k\ne 0$ may be defined in Fourier space as

$$(\xi_t)_k = (\zeta_t)_{-k}\coth kH. \tag{10}$$

Finally, the new form of bottom boundary condition, which assumes vanishing of vertical velocity at the bottom, is given

$$\Phi_\zeta(\xi,\zeta=-H,\tau) = 0. \tag{11}$$

The solution of Laplace Eq. (5) with boundary condition (11) readily yields the Fourier expansion, which reduces the system