Ocean Engineering 125 (2016) 272–284

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

A general frequency-domain dynamic analysis algorithm for offshore structures with asymmetric matrices

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ARTICLE INFO

Article history:
Received 31 March 2016
Received in revised form
18 July 2016
Accepted 20 August 2016
Available online 30 August 2016

Keywords:
Nonclassical structure
Asymmetric damping
Response estimation
Frequency domain
Nonzero initial conditions
Tension leg platform

ABSTRACT

Asymmetric system matrices, especially the asymmetric damping matrix, must be considered because of the usual drastic variations between the energy absorption rates of materials in different parts of a structure. This paper proposes a general frequency-domain response estimation method for incorporating possibly asymmetric mass, stiffness and damping matrices in engineering. One achievement is that a general frequency response function (GFRF) is defined by estimating the coefficients, poles and zeros of the structure, rather than by using the eigenvalues and eigenvectors. The second is that nonzero initial conditions can also be considered in the frequency domain. Three examples are employed: a system with four degrees of freedom, a frame structure, and a tension leg platform. One can conclude the following: (1) the proposed method can provide more accurate results for estimating FRFs of the system with asymmetric matrices; (2) traditional frequency-domain method can be regarded as a special case of the approach, just aiming at estimating steady-state responses of the system with symmetric matrices; (3) transient responses of a system with asymmetric matrices can also be estimated properly and with a good efficiency, based on the discretization of the reconstructed external loadings.

1. Introduction

Non-symmetric/asymmetric second-order systems are often found in a number of engineering contexts, such as in the study of rotor dynamic models and long-span bridges, as well as in the behavior of structures in fluids and in aircraft flutter. Considering the fact that mass and stiffness properties can typically be accurately simulated using mathematical modeling techniques, such as the finite element method, while the mechanisms of energy dissipation remain notoriously difficult to be represented numerically. Moreover, the specification of the nature and magnitude of damping in structures is unavoidably subjected to considerable uncertainty; the normally used classical damping is only a justifiable approximation in many practical applications.

When damping is of the form specified by Caughey and O'Kelly (1965), which is referred to as classical damping or proportional damping, the natural modes of the system have real values and are identical to those of the associated undamped system. Systems satisfying this condition are said to be classically damped, and the response of classically damped systems is obtained by the modal superposition method, i.e., the traditional frequency-domain dynamic analysis method, which is generally recognized as a powerful method that can be used to evaluate the dynamic response of viscously damped linear structural systems. This method expresses the response of a system with multiple degrees of freedom (DOFs) as a linear combination of its corresponding modal responses; widespread applications can be found in civil engineering because of its conceptual simplicity and ease of application as well as the insight it provides into the action of the system. The first step of the procedure involves forming a discretized mathematical idealization and evaluating the mass, stiffness matrices and effective external force vector in the discretized coordinates. Then, after the undamped vibration mode shapes are evaluated, the equations of motion are uncoupled by transformation to these modal coordinates, and the dynamic response is computed separately for each coordinate. A basic assumption of this method, which has been verified by field measurements for many types of structures, is that damping causes no significant coupling of the modal response equations. However, the precondition for the validation of the independent modal damping is that there should be a reasonable degree of homogeneity in the energy loss mechanisms throughout the structure. Actually, classical damping is typically a rare occurrence in practice rather than a common one: drastic variations in the energy absorption rates in different parts

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http://dx.doi.org/10.1016/j.oceaneng.2016.08.024
0029-8018/

of the structure will cause the distribution of the damping forces to be quite different from those of the elastic and inertial forces during free vibrations because most large-scale, real-life dynamic systems are typically composed of different subcomponents. Even if we were to ascribe a viscous damping character to each of these subcomponents, the final damping matrix **C**, which is constructed through a finite element model for the whole system, would generally be of the nonclassical type, which induces coupling between modal coordinates and thus invalidates the use of the standard mode superposition method of response analysis.

Most of the previous investigators have focused on diagonalizing the damping matrix by ignoring the off-diagonal elements so that the standard modal superposition method can be used. Shahruz and Langari (1991) studied the errors introduced in the system response from the decoupling achieved by simply ignoring the off-diagonal terms of the damping matrix. They specified the conditions under which the solution of this approximately decoupled system was close to the solution of the coupled system. Shahruz and Packard (1991) further investigated the errors that can arise in lightly damped systems under harmonic excitations when some of the undamped natural frequencies of the system are close to the excitation frequency. Felszeghy (1993) presented a method of searching for another coordinate system in the neighbourhood of the normal coordinate system such that the removal of the coupling terms in the equations of motion produces a local minimum of the norm of the relative error in the new coordinate system. Udwadia and Kumar (1994) proposed a variety of new computationally efficient iterative methods to determine the response of such systems; their results showed vastly improved rates of convergence. However, a recent work by Morzfeld et al. (2009) demonstrated that over a finite range, errors due to the decoupling approximation can increase monotonically at any specified rate, whereas the modal damping matrix becomes more diagonally dominant as its off-diagonal elements continuously decrease in magnitude.

The key challenge for estimating the dynamic responses of nonclassical structures is that the frequency response functions (FRFs) cannot be obtained using the estimated eigenvalues and the corresponding eigenvectors, which is the method used in the traditional frequency domain, because the mass matrix **M**, the stiffness matrix **K** and the damping matrix **C** cannot be simultaneously diagonalized in the modal coordinates. To estimate eigenvectors of the nonclassical structures, Adhikari and Friswell (2001) extended the modal method to the first-order and second-order derivatives of the eigensolutions of the asymmetric damped systems. To reduce the number of eigenvectors needed to compute the derivative of each eigenvector, Zeng (1995) presented a modified modal method for complex eigenvectors in symmetric damped systems. Later, Moon et al. (2004) extended the modified modal method to general asymmetric damped systems. Guedria et al. (2007) extended Nelson's method to second-order eigenvector derivatives for symmetric and asymmetric damped systems. Lee et al. (1999) developed an algebraic method with an asymmetric coefficient matrix for first-order and second-order equations of motion. Chouchane et al. (2007) extended their method to the second-order derivatives of eigensolutions and noted that their method can be extended to compute the higher order eigensolution derivatives. Recently, Chen and Tan (2012) proposed a new algebraic method to compute the eigensolution variability of asymmetric damped systems. Some weight constants were introduced to make the proposed method well-conditioned, and the results showed that the method was very compact and highly efficient. Lázaro (2016) proposed a closed-form expression for complex eigenvalues in non-proportional viscously damped system. It has been shown that the proposed formula estimated the exact values up to the second order approximation in terms of the modal damping matrix. Because the developments were made on the basis of a small damping assumption, the accuracy became poorer as the level of damping increased.

In this paper, which does not use the decoupling approximation or eigenvector estimation techniques discussed above, we estimate a structure's poles, zeros and their coefficients to calculate the general frequency response functions (GFRFs) of a nonclassical structure. The biggest advantage of this approach is that not only the coupling problem of damping but also that of the stiffness and even of the mass matrix can be solved simultaneously, which also means that the steady-state responses of the structure can be estimated as the traditional frequency-domain method does simply by replacing their FRFs with the GFRFs from our approach. Furthermore, we will also extend this new frequency-domain response estimation method to simultaneously estimating transient and steady-state responses based on other works (Liu et al., 2015, 2016). Three numerical examples demonstrate the proposed method: one is a lumped system with four DOFs, the second is a multiple DOF frame structure, and the third is a triangle tension leg platform. Comparisons of the estimated responses from the proposed method with those from the time-domain and traditional frequency-domain method will be included as well.

2. Preliminaries

2.1. Modal damping

The type of system damping (Craig and Kurdila, 2006) that is most frequently used in structural dynamics computations is referred to in the literature as Rayleigh damping and is defined by

$$\mathbf{C} = a_0\mathbf{M} + a_1\mathbf{K} \tag{1}$$

where **M** and **K** are the system mass and stiffness matrices, and a_0 and a_1 are two coefficients that are typically estimated by choosing the system damping factor ζ_r and the natural frequency ω_r for the two modes:

$$\zeta_r = \frac{1}{2}\left(\frac{a_0}{\omega_r} + a_1\omega_r\right) \tag{2}$$

Then, the modal damping, which is often denoted by a superscript *, is a diagonal matrix given by the following:

$$\mathbf{C}^* = \mathbf{\Phi}^T\mathbf{C}\mathbf{\Phi} = \mathrm{diag}(\mathbf{C}_r^*) = \mathrm{diag}(2\zeta_r\omega_r\mathbf{M}_r^*) \tag{3}$$

where $\mathbf{M}_r^*$ is the modal mass matrix given by $\mathbf{M}_r^* = \mathbf{\Phi}_r^T\mathbf{M}\mathbf{\Phi}_r$, $\mathbf{\Phi}_r$ is the rth mode of the undamped system, and the superscript ^{T} represents the transpose of a vector.

2.2. Steady-state response estimation using the traditional frequency-domain method

The equations of motion of a linear multiple DOFs system (Clough and Penzien, 1993) are

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \tag{4}$$

where $\mathbf{f}(t)$ represents the subjected loads and $\ddot{\mathbf{x}}(t)$, $\dot{\mathbf{x}}(t)$ and $\mathbf{x}(t)$ are the accelerations, velocities and displacements, respectively.

Through use of the normal modes of the undamped system and the assumption of modal damping (Eq. (3)), the equations above can be transformed into a set of uncoupled modal equations of motion:

$$\ddot{\eta}_r(t) + 2\zeta_r\omega_r\dot{\eta}_r(t) + \omega_r^2\eta_r(t) = \frac{1}{\mathbf{M}_r^*}\mathbf{\Phi}_r^T\mathbf{f}(t) \quad r = 1, 2, \ldots, N \tag{5}$$