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Trend analysis of the power law process using Expectation–Maximization algorithm for data censored by inspection intervals

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ABSTRACT

Trend analysis is a common statistical method used to investigate the operation and changes of a repairable system over time. This method takes historical failure data of a system or a group of similar systems and determines whether the recurrent failures exhibit an increasing or decreasing trend. Most trend analysis methods proposed in the literature assume that the failure times are known, so the failure data is statistically complete; however, in many situations, such as hidden failures, failure times are subject to censoring. In this paper we assume that the failure process of a group of similar independent repairable units follows a non-homogenous Poisson process with a power law intensity function. Moreover, the failure data are subject to left, interval and right censoring. The paper proposes using the likelihood ratio test to check for trends in the failure data. It uses the Expectation–Maximization (EM) algorithm to find the parameters, which maximize the data likelihood in the case of null and alternative hypotheses. A recursive procedure is used to solve the main technical problem of calculating the expected values in the Expectation step. The proposed method is applied to a hospital's maintenance data for trend analysis of the components of a general infusion pump.

1. Introduction

Trend analysis is a common statistical method used to investigate the operation and changes in a repairable system over time. This method takes historical failure data of a system or a group of similar systems and determines whether the recurrent failures exhibit an increasing or decreasing trend. Both graphical methods and trend tests are used for trend analysis. The latter are statistical tests for the null hypothesis that the failure process follows a homogenous Poisson process (HPP) [1].

Crow/AMSAA test [2,3] assumes that the failure process of a repairable system has a Weibull intensity function and finds the maximum likelihood estimates of the parameters. The shape parameter is then used as an indicator for a growth or deterioration in the system reliability. The Laplace and the Military Handbook tests [4] are used to test whether data follow a HPP. Kvaloy and Lindqvist [5] propose the Anderson–Darling trend test for a NHPP with a bathtub shaped intensity function based on the Anderson–Darling statistic test [6]. The Lewis–Robinson [7] is a modification of the Laplace test and is used as a general test to detect trend departures from a general renewal process [8]. The Mann test [9] corresponds to a renewal process null hypothesis and the monotonic trend as the alternative. Kvaloy et al. [10] declare that the Mann test is a more powerful test against decreasing trend. The MIL-HDBK-189 [11,12] is used for testing a NHPP with a power law intensity function. Caroni [13] modifies the trend tests introduced by Kvaloy et al. [10] for data that end with the last of a random number of failures within a predetermined observation period. Louit et al. [14] review several tests available to assess the existence of trends, and proposes a practical procedure to discriminate between the use of statistical distributions to represent the time to failure. Regattieri et al. [15] introduce a framework defining a general approach for failure process modeling, and demonstrate the application of the proposed framework in a light commercial vehicle manufacturing system.

Most trend tests [4,16] assume that failure times are known, so the failure data are complete. Currently in the literature, except for right censoring, there is no available method for estimating the parameters of a non-homogeneous Poisson process (NHPP), which incorporates left and interval censored failure data if repairs are not instantaneous or not performed immediately. However, it is expected that the data received from industry include missing and incomplete information. Sometimes a particular type of data of interest is not measured at all, or if measured may be incomplete or unreliable. For example, hidden failures which make up to 40% of all failure modes of a complex industrial system [17] are not evident to operators and remain dormant

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until they are rectified at scheduled inspections. In this case, the times of hidden failures are either left or interval censored, and the censoring interval is the interval between two consecutive inspections.

Weibull or power law [2] and log linear [18] intensity functions are two common models used to describe recurrent even data underlying NHPP. The maximum likelihood estimates of the parameters are obtained for two intensity parameters [12,19]. In this paper, we assume that the failure process follows an NHPP with a power law intensity function. We use the Expectation–Maximization (EM) algorithm to estimate parameters of the power law process and propose a recursive method to calculate the expected values in the EM Expectation step. In the Maximization step, we use the Newton–Raphson method. We then apply the general likelihood ratio test to test HPP against NHPP alternative [12]. In some practical situations, the mid-points of the censoring intervals may be considered as the failure times and used to estimate the parameters of the failure process and to perform trend test analysis. If the inspection intervals are not short, this approximation of actual failure times can be inaccurate.

A detailed problem description is given in Section 2. Section 3 describes the EM algorithm and proposed trend test method, while Section 4 includes numerical examples using adapted data from the case study [20]. Section 5 presents our conclusions.

2. Problem definition

Consider a repairable system or a unit subject to censored failure times. For example, an audible component in an infusion pump in a hospital is used to communicate with operators and inform them of the status of the patient to whom the device is attached. When the level of liquid delivered to a patient is reduced to a certain level, the audible component starts sending warning alarms. If the component fails, the pump can still function, but the patient's health risk increases if the operator does not take any action. The failures of an audible component are hidden and are only rectified at periodic inspections of the system. Because the failure times are censored, the times spent in a failed state are unknown and so also the total operating time of the component. Another example is a measuring head in a thickness gage used in the steel industry. Any change in the measuring head can produce a less accurate measurement of a steel strip and hence should be considered a failure. The exact failure (deforming) times may not be known and are censored. Our main objective is to develop a method to perform a trend analysis of censored failure data for a unit or a group of similar units. We make the following assumptions for the units under study:

(i) Failures of the units follow an NHPP with a power law intensity function $\lambda(x)=\beta e^{\alpha}x^{\beta-1}$.
(ii) The units are inspected periodically at times $k\tau$, $(k=1,2,\ldots)$ over a life cycle of length T.
(iii) Failures are only rectified at inspections.
(iv) A unit is not aging when not operating, i.e. from failure to inspection.
(v) At the beginning, a unit may have an initial age y_0 with $y_0=0$ if it starts as good as new.
(vi) The unit is minimally repaired at inspection, if found failed.

It should be noted that more common form to present a power law is $\lambda(x)=(\beta/\eta)(x/\eta)^{\beta-1}$ or $\Lambda(x)=\int_0^x \lambda(t)dt=(x/\eta)^{\beta}$; however, for convenience of maximum likelihood estimation we use $\lambda(x)=e^{\alpha}\beta x^{\beta-1}$ and $\Lambda(x)=e^{\alpha}x^{\beta}$.

Fig. 1 shows an example of the failures of a unit with initial age y_0. The first failure takes place in the first inspection interval, i.e. in $(0,\tau]$, but the failure time is not known (left censored by τ). The unit does not age in the period between the failure time and the inspection time. Random variable X_1, $0\le X_1\le\tau$, denotes the surviving time of the unit in the first inspection interval. The unit does not fail in the second inspection interval, i.e. in $(\tau,2\tau]$, and the second failure occurs between the second and third inspection. The second surviving time of the unit, X_2, $\tau\le X_2\le 2\tau$ is interval censored. Thus, the censoring intervals are the intervals between two consecutive inspections. Since we assume the minimal repairs, distribution of X_2 depends on the age of the unit at time τ, and this age is X_1. Similar description applies to the other possible failures and the surviving times $X_3,\ldots, X_r$.

Our first goal is to estimate parameters of the NHPP from censored data on $X_1,X_2,\ldots$ using the maximum likelihood method and to test for possible trend in the data using the likelihood ratio test. Because the random variables $X_1,X_2,\ldots$ are dependent, our problem is complex. Note that in our approach, inspection times need not be periodic; nonequal inspection intervals can also be used.

3. Parameter estimation and trend analysis

3.1. Trend test

We propose to use the likelihood ratio test [19,21] to test for possible trend in failure times, with null (H_0) and alternative (H_1) hypotheses, as follows:

H_0: homogenous Poisson process ($\beta=1$) (no trend)
H_1: non-homogenous Poisson process ($\beta\neq 1$) (trend)

Let $L(\alpha,\beta)$ is the likelihood of the observed data, which is given in detail by Eq. (4). Under H_0, parameter α should be estimated using maximum likelihood assuming that $\beta=1$. Let then, $L_0=L(\hat{\alpha},\beta=1)$.

Under H_1, both parameters are estimated by maximizing the likelihood, and let $L_1=L(\hat{\alpha},\hat{\beta})$. The likelihood ratio is then defined as the statistic $\chi^2=-2\ln(L_0/L_1)$ which under H_0 follows χ^2 distribution with 1 degree of freedom (L_0 has 1 parameter less than L_1). The null hypothesis is rejected if χ^2 is greater than the appropriate critical value, that is $\chi^2>\chi^2_{1,\alpha}$, where $P(\chi^2_1>\chi^2_{1,\alpha})=\alpha$. If $\chi^2\le\chi^2_{1,\alpha}$ we do not reject the null hypothesis and conclude that the failure process does not exhibit a visible trend. If $\chi^2>\chi^2_{1,\alpha}$ and $\hat{\beta}>1$, there is an indication of increasing trend in the failure process, i.e. the units are degrading over time. If $\chi^2>\chi^2_{1,\alpha}$ and

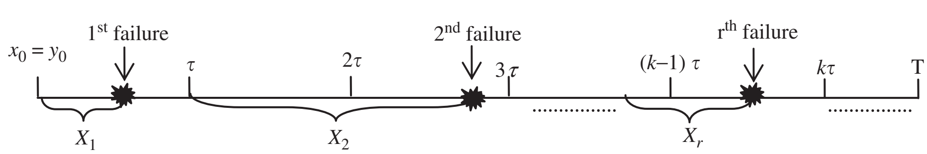

Fig. 1. Initial age y_0 and failure times of a unit.