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# Application of generalized finite difference method to propagation of nonlinear water waves in numerical wave flume

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ABSTRACT

In this paper, a numerical wave flume is formed by combining the generalized finite difference method (GFDM), the Runge–Kutta method, the semi-Lagrangian technique, the ramping function and the sponge layer to efficiently and accurately analyze the propagation of nonlinear water waves. On the basis of potential flow, the mathematical description of wave propagation is a time-dependent boundary value problem, governed by a Laplace equation for velocity potential and two nonlinear free-surface boundary conditions. The incident waves are introduced by imposing horizontal velocity along upstream boundary, as a sponge layer is placed at the end of flume to absorb wave energy and avoid any reflection of waves. The GFDM, a newly-developed meshless numerical method, and the second-order Runge–Kutta method were, respectively, adopted for spatial and temporal discretizations of the moving-boundary problems. The GFDM, which is truly free from mesh generation and numerical quadrature, is easy-to-program, straightforward and efficient, especially for moving-boundary problems. Four numerical examples are adopted in this paper to validate the stability, the efficiency and the accuracy of the proposed meshless numerical wave flume. The GFDM results were compared with other numerical solutions and experimental data to verify the merits and robustness of the proposed meshless numerical model.


## 1. Introduction

The propagation of nonlinear water waves along free surface plays an important role in the fields of costal and ocean engineering. When the water waves are reaching a near-shore zone, they may result in erosion and endanger the safety of coastal structures. In addition, recent developments of mechanical design for acquiring clean ocean energy from wave, tide and current (Rahmati and Aggidis, 2016) are of primary importance due to possible shortage of energy and electricity in the incoming future. Furthermore, the interactions of water waves and ships motion (Faltinsen, 1977; Fontaine and Tulin, 2001; Landrini et al., 2012; Maruo and Song, 1994; Tulin and Wu, 1996; Vinje and Brevig, 1981) are quite important and interesting in naval engineering. Hence, to discover the underlying physics of water-waves propagation in the ocean is essential and urgent to academic and industrial communities. Owing to the rapid developments of computer software and hardware in the past half century, numerical simulation becomes a good alternative in comparison with experimental researches and theoretical studies for water-waves problems. Therefore, in this paper, based on a newly-developed meshless method, an efficient, accurate and stable numerical scheme was proposed for the propagation of nonlinear water waves in a two-dimensional numerical wave flume.

In the past decades, various numerical models (Gotoh et al., 2013) have been proposed to simulate the generation and propagation of non-linear water waves. For example, Li (2008) used the projection method and sigma transformation for analyzing the propagation of regular and irregular water waves, governed by the Navier–Stokes equations. Since the nonlinearity and complexity of the Navier–Stokes equations may induce difficulties in numerical models, some simplified mathematical descriptions for flow field are proposed under reasonable assumptions. One of the main research directions is to adopt the theorem of potential flow, so the flow fields of water-waves propagation in some studies are assumed to be inviscid, irrotational and incompressible. For potential flow, the governing equation for velocity potential is the well-known Laplace equation, which is a linear second-order partial differential equation. Using the assumption of potential flow, Koo and Kim (2004, 2007) used the boundary element method (BEM) and the fourth-order Runge–Kutta method to simulate the propagation of nonlinear water waves and the interaction between waves and fully-floating bodies, while Christou et al. (2008) adopted the multiple-fluxes BEM to model the

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interactions between non-linear water waves and rectangular submerged breakwaters. In addition, under the assumptions of potential flow there are various numerical schemes (Contento, 2000; Contento et al., 2001; Madsen, 1971; Ryu et al., 2003; Senturk, 2011; Wu et al., 2006, 2008; Zhang et al., 2006), which have been proposed to analyze problems of water-waves propagation. Following these researches for water waves over past half century, the flow field in the numerical wave flume in this paper is assumed to be potential flow. From some of the important researches (Grilli et al., 1989; Koo and Kim, 2004, 2007; Longuet-Higgins and Cokelet, 1976; Wang et al., 1995), it is noticed that to study the propagation of water waves by the BEM is a very classical and popular way. Though to use the BEM can reduce the dimensionality of problem and simplify the numerical discretization, the troublesome problems of full matrix, singular/hyper-singular integrals and generation of line/surface mesh in the BEM are still needed to be relieved. On the other hand, the meshless numerical wave flume, proposed in this paper, is based on the generalized finite difference method (GFDM). Therefore, the proposed numerical flume can get rid of time-consuming tasks of meshing and numerical quadrature. Although the dimensionality of problem cannot be simplified, the conception of localization, which means the star in the GFDM, can result in a sparse system of linear algebraic equations, which can be efficiently resolved.

In the past few decades, numerous so-called meshless methods, which are free from time-consuming mesh generation, have been proposed to analyze mathematical problems and engineering applications, such as the boundary knot method (Chen, 2002), the singular boundary method (Chen et al., 2009), the local radial basis function collocation method (Chan and Fan, 2013; Senturk, 2011), the GFDM (Benito et al., 2001, 2007; Chan et al., 2013; Fan et al., 2014, 2015; Gavete et al., 2003; Li et al., 2014; Urena et al., 2012; Zhang et al., 2016), etc. Among them, the GFDM is one of the most-promising newly-developed domain-type meshless methods. The explicit formulas of the GFDM to express the spatial derivatives by using the moving-least-squares approach were proposed by Benito et al. (2001). They also systematically examined the influence of some factors in the GFDM on the numerical accuracy by numerical experiments. The spatial derivatives can be expressed as linear combinations of nearby function values with different weightings. Since the GFDM is evolved from classical finite difference method, to enforce the satisfactions of governing equation at every interior node and boundary condition at every boundary node can accurately and stably acquire numerical solutions via a collocation approach.

Recently, the GFDM has been successfully applied to accurately and efficiently solve parabolic and hyperbolic partial differential equations (Benito et al., 2007) as well as higher-order partial differential equations (Urena et al., 2012). Additionally, Fan et al. (2014, 2015) analyzed two-dimensional inverse problems in a stable manner by adopting the GFDM, while Li et al. (2014) used the GFDM to accurately solve the problems of density-driven groundwater flow. Zhang et al. (2016) used the GFDM and the explicit Euler method for numerical solutions of moving-boundary problems of sloshing phenomenon. From the above descriptions, it is worth mentioning that the newly-developed GFDM remained the advantages from both of the mesh-based methods and the meshless methods, so it has great potential to be applied to various engineering problems. In this paper, the GFDM is adopted for spatial discretization of water-waves propagation in a numerical wave flume.

In our previous study (Zhang et al., 2016), we considered only rectangular flat-bottom domains for sloshing problems without inlet and outlet boundary. From some comparisons, it is found that the numerical scheme for temporal discretization should be substantially improved to enhance the numerical stability and enlarge the time increment, though the combination of the GFDM and the explicit Euler method can accurately simulate the moving-boundary problems of sloshing phenomenon. In comparison with the previous GFDM study of sloshing phenomenon (Zhang et al., 2016), in the present paper the second-order Runge–Kutta method was adopted for temporal discretization of the moving-boundary problems of water-waves propagation. In addition, different variations of seabed were considered to examine the interaction between water waves and irregular bottom profiles. Moreover, the wavemaker and the sponge layer are introduced in the inlet and outlet boundary sections, respectively. It can be noticed that the proposed meshless wave flume is an extension research of the previous GFDM study (Zhang et al., 2016). In this paper, the treatments of inlet and outlet boundary, the time integrator with better stability and the study of interactions between nonlinear waves and seabed are all considered. In the proposed numerical scheme, the method for temporal discretization and the semi-Lagrangian approach are used to update the spatial position of every node and acquire the potential along free surface. Once the space coordinate of every node is updated, the GFDM is employed to efficiently deal with two-dimensional Laplace problem at the present time step. The above-described procedures will be repeated until the terminal time is reached.

An accurate, efficient and stable numerical scheme is proposed in this paper to simulate the propagation of nonlinear water waves passed over flat and irregular bottom topography. The GFDM and the second-order Runge–Kutta method are adopted for spatial and temporal discretizations, while the semi-Lagrangian approach is used to update the spatial coordinates of every node. The theory of wavemaker for incident waves and a sponge layer for outgoing waves are also combined in the proposed scheme. The mathematical descriptions and the numerical methods are elaborated in the following sections. Four numerical examples are provided to examine the merits of the proposed meshless scheme. In the last section, some conclusions and discussions are drawn according to the provided results and comparisons.

## 2. Governing equation and boundary conditions

### 2.1. Governing equation, bottom boundary condition and free-surface boundary conditions

In this paper, a two-dimensional water-waves propagation problem in a rectangular numerical flume is considered. The Cartesian coordinate system $(x, z)$ is attached to this flume and the origin is located at the left bottom corner of this flume, which is demonstrated in Fig. 1. The depth of initial still water and the length of flume are denoted by $h$ and $b$, also depicted in Fig. 1. The flow field in the proposed numerical flume is assumed to be potential flow, so the flow field is governed by the Laplace equation for velocity potential,

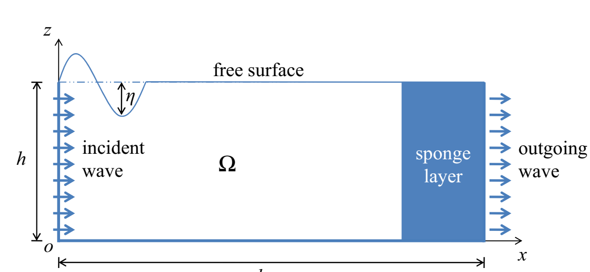


**Fig. 1.** The schematic diagram of numerical wave flume.