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Automatic generation of dynamic models of cables

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ABSTRACT

A new theory for the dynamic modeling of cables is presented in this paper, focusing on underwater applications. The main idea is to approximate the continuous flexibility of the cable by several rigid links connected by fictitious elastic joints, allowing three movements: elevation, azimuth and torsion. The Lagrangian of the system is written in a compact form and can be generated for any number of links chosen to represent the structural dynamics. The application of Euler–Lagrange equations allows to obtain the dynamic model, which in this article was developed analytically for the cases of 2, 3 and 4 links. The dynamic model's equations grow significantly with the growth of the number of links and a detailed analysis of this growth enabled the proposition of generic algorithms for the automatic generation of the vectors and matrices elements, for whatever number of considered links. This theory was proposed considering a cable fixed at one end and free at the other, containing a terminal load. However, it can be easily adapted to flexible structures fixed at both ends and for applications underwater or out of water. The generic algorithms proposed in this article allow fast and automatic retrieval of dynamic models of cables, considering a large number of links to represent the structural flexibility, that would be unfeasible to obtain manually.

1. Introduction

Dynamic modeling of flexible structures such as a cable is a complicated task, mainly due to the several degrees of freedom needed to represent a very complex dynamics, especially when considering movements in 3D space. Many applications involving cables dynamics occur in the underwater environment. The knowledge of the dynamic structural model such as risers, mooring lines, towing cables, etc., can be of great importance for the offshore oil industry, for example (see Fig. 1).

Most studies found in the literature address the modeling of these structures using Finite Element Methods (Wang et al., 1998; Gosling and Korban, 2001). Other authors also used finite element methods for the structural dynamic analysis of flexible cables (Buckam et al., 2004; Srinil et al., 2007; Yoon et al., 2008). Sun et al. (2011) introduced a finite element method to modeling a cable towed body.

Some authors have developed their works performing a cable static analysis (Hover et al., 1994; Matulea et al., 2008; Wang et al., 2008), using the method of finite differences. A static analysis of two-dimensional cables is also made in Dreyer and Van Vuuren (1999), using numerical solution of both continuous and discrete models. Discrete approach was used specially in static analysis: Raman-Nair and Williams (2005) have used a discrete model to reproduce structural forces acting into a flexible marine riser under effects of flow and pressure of fluid within the riser; Zhu et al. (2008) proposed a discrete model to determine the forces that an umbilical cable exerts on a ROV (Remotely Operated Vehicle).

When the discrete formalism is used in dynamic modeling, usually lumped mass approach is applied, considering the dynamics evolving in a single plane. A simulation of cable dynamics for kites was made by Breukels and Ockels (2007) considering each link with one degree of freedom mass spring damper model and in that case, the flexible structure's motion was restricted in a single vertical plane. Hall and Goupee (2015) used a lumped mass approach to modeling a mooring line and validated the simulations with an offshore wind turbine test data.

Finite differences are widely used in cable modeling. Lacarbonara and Pacitti (2008) used finite differences to modeling cables suffering axis stretching and flexural curvature. In Srivastava et al. (2011) a three-dimensional model of underwater towed cable is studied and governing equations are solved by using a central finite-difference method. Matulea et al. (2014) used finite differences, first to determine the static equilibrium configuration of the riser, and then to find its dynamic response around the formerly computed static configuration, considering the flexible structure restrict to the vertical plane. Lee et al. (2015) applied finite difference method with lumped mass to modeling a flexible pipe. Zhang and Li (2015) analyzed axial dynamic stress response of deep water risers and a Linear Quadratic Gaussian control was

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proposed to deal with this problem.

In short, most of the articles that deal with cables treat the problem as restricted to a single plane using finite elements or finite differences for the dynamic model. Other works focus interest in static analysis of axial forces on the cable. Gobat (2000) in his thesis provides details about these main methods used in cable dynamics.

This paper introduces a new method to automatically obtain vectors and matrices elements of a cable dynamic model. We use the discrete formalism to represent the continuous flexibility from a chain of rigid links connected by fictitious elastic joints. Each joint allows three elastic movements: elevation, azimuth and torsion. We take as a basis the work of Gomes et al. (2006), wherein the discrete formalism was used to model a robotic manipulator with a single flexible link. In that case, the flexibility occurred on a single plane and each joint had one degree of freedom. Based on this work, Pereira et al. (2012) developed the analytical modeling of a cable considering 3 links, but with spatial flexibility, i.e. the discrete formalism was used without the motion being restricted to a single plane. The present article complete in a definite way that proposed by Pereira et al. (2012), since, from the Lagrangian written in a generic way (for any number of links) proposes generic algorithms to determine automatically the dynamic model, for any number of links chosen to represent the continuous flexibility. Automatic retrieval models is very important due to the great complexity of the equations that turn unfeasible obtaining these models manually through the application of the Euler–Lagrange equations. Generic algorithms are the great innovation and contribution of this article.

2. Fundamentals of the proposed theory

In this work it is considered a cylindrical cable with constant radius, fixed at one extremity (fixed base) and free at the other, where there is a terminal load m_c. The basic principle of this modeling theory is to approximate the continuous flexibility by a discrete equivalent one, consisting of rigid links connected by flexible fictitious joints, as showed in Fig. 2. Each fictitious elastic joint allows three movements: elevation, azimuth and torsion. Therefore, this dynamic system has $3n$ degrees of freedom when considering n links. In each fictitious joint is positioned a reference frame, as shown in Fig. 3 for the first two systems. The first is an inertial system ($X_0\,Y_0\,Z_0$). It was adopted the following convention for reference systems: all Z axes point to the center of the Earth and thus, the XY axes form horizontal planes. The Y_i axes are parallel to the projection of the link i on the $X_{i-1}Y_{i-1}$ plane, as showed in Figs. 3 and 4. For instance, Y_1 is parallel to r in Fig. 3. Fig. 4 also shows the three angular positions coordinates of the first joint and the three others of the second fictitious joint.

It is very simple to find a homogeneous transformation matrix between two consecutive reference systems. For example, the homogeneous matrix that relates $X_0Y_0Z_0$ and $X_1Y_1Z_1$ systems has the form:

$$H_{01}=\begin{bmatrix}\cos\theta_{1a} & \sin\theta_{1a} & 0 & l_1\sin\theta_{1e}\sin\theta_{1a}\\ -\sin\theta_{1a} & \cos\theta_{1a} & 0 & l_1\sin\theta_{1e}\cos\theta_{1a}\\ 0 & 0 & 1 & l_1\cos\theta_{1a}\\ 0 & 0 & 0 & 1\end{bmatrix} \tag{1}$$

The products between successive homogeneous matrices generate another homogeneous matrix that can relate any mobile reference system to the base inertial system of the structure. Thus, the spatial position of the center of mass of any link in the inertial frame may be determined as functions of the lengths of the links and the angular position coordinates, as specified below $(k = 1, \ldots, n)$:

$$\begin{cases} x_k = \dfrac{l_k}{2}\sin\theta_{ke}\sin\left(\sum_{i=1}^{k}\theta_{ia}\right)+\sum_{j=1}^{k-1}\left[l_j\sin\theta_{je}\sin\left(\sum_{i=1}^{j}\theta_{ia}\right)\right]\\ y_k = \dfrac{l_k}{2}\sin\theta_{ke}\cos\left(\sum_{i=1}^{k}\theta_{ia}\right)+\sum_{j=1}^{k-1}\left[l_j\sin\theta_{je}\cos\left(\sum_{i=1}^{j}\theta_{ia}\right)\right]\\ z_k = \dfrac{l_k}{2}\cos\theta_{ke}+\sum_{j=1}^{k-1}l_j\cos\theta_{je}\end{cases} \tag{2}$$

Arising from the same formalism, spatial coordinates of the terminal load (written in the inertial frame) have the form:

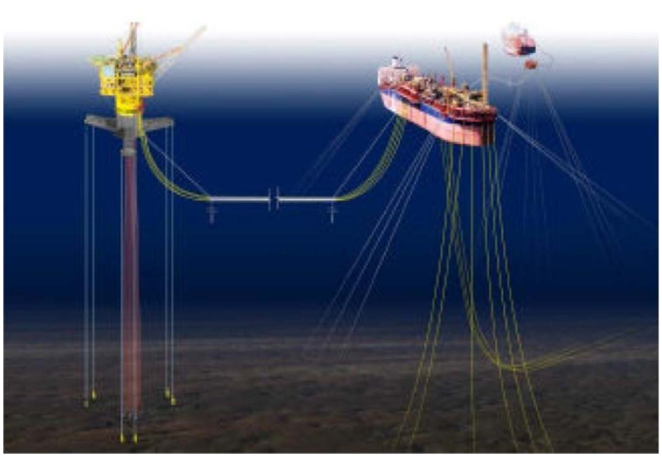

Fig. 1. Cables in underwater applications (font: http://diariodopresal.wordpress.com/petroleo-e-gas).

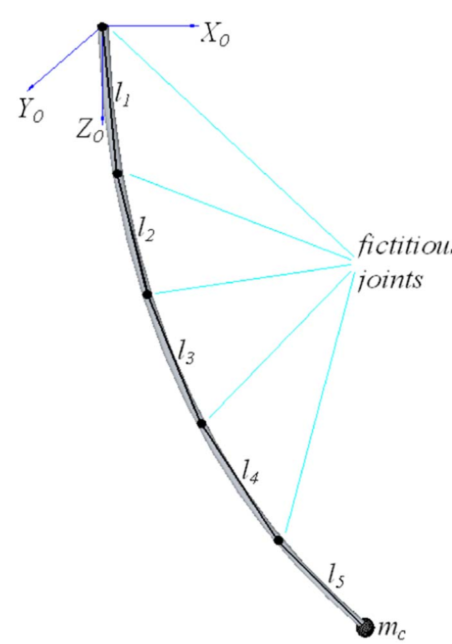

Fig. 2. Continuous flexibility and its discrete approximation.