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Uncertainty analysis procedure for the ship inclining experiment

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ABSTRACT

The inclining experiment is typically performed for all new-build ships and after any major refit. The purpose of the inclining experiment is to establish the vertical distance of the centre-of-mass of the ship above its keel in the lightship condition. This value is then taken as the point of reference when loading the ship, for establishing the 'in-service' stability, throughout the life of the ship. Experimental uncertainty analysis is commonly utilised in hydrodynamic testing to establish the uncertainty in a result as a function of the input variables. This can in turn be utilised to establish an interval about the result that may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurement. This paper provides a methodology for calculating a confidence interval for the location of the centre-of-mass of a ship from an inclining experiment; and ultimately, in any load condition.

The uncertainty compared to an assumed metacentric height of 0.15 m is provided for four classes of ship: buoy tender 0.15 ± 0.15 m ($\pm$100%); super yacht 0.150 ± 0.033 m ($\pm$22.0%); supply ship 0.150 ± 0.047 m ($\pm$31.3%), container ship 0.150 ± 0.029 m ($\pm$19.3%), ropax 0.150 ± 0.077 m ($\pm$100%).

1. Aims and objectives

The aim is to establish procedures for identifying the experimental uncertainty in the estimate of the centre-of-mass height above the keel (referred to as $\overline{KG}$) by method of an inclining experiment (IE).

The first objective is to give procedures for performing a pre-test analysis that can be employed to identify the best course of action for reducing the experimental uncertainty. The second objective is to give procedures for performing a post-test analysis that can be employed to identify a confidence interval for the resulting estimate of $\overline{KG}$.

2. Background

The IE is a required procedure [unless exceptions apply; see IMO, 2008] for all new-build ships and after any major refit. The purpose of the IE is to establish $\overline{KG}$, in the lightship condition. This value is then taken as the point of reference when loading the ship, for establishing the 'in-service' $\overline{KG}$, throughout the life of the ship. An accurate estimate of the limiting $\overline{KG}$ is absolutely necessary for the safe operation of the ship, so as to ensure adequate stability. Clearly, this is dependent on an accurate estimate of the lightship $\overline{KG}$ obtained from the IE.

While typically all attempts are made to conduct the IE in a manner that minimises the introduction of error, many potential sources of error exist. For example, all attempts are made to remove the influence of fluid free-surface effects, by emptying or pressing-full all tanks. Any suspended loads are secured or removed and anything that may move is removed or made secure. Similarly, all attempts are made to conduct the IE in calm conditions, when the effect of wind, waves, current and the wash from passing ships is minimised.

Notwithstanding all attempts to minimise errors, sources of uncertainty will always be present – uncertainty being different from error. Due to the stochastic nature of the world, all input variable measurements are only known with limited accuracy. The uncertainty in the results (in this case the estimate of $\overline{KG}$) is dependent on the magnitude of the uncertainties of each input variable and on the particular sensitivity of the results to each input, which is dependent on the form of the data reduction equations.

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2.1. Overview of the inclining experiment

Explanations of the procedure for an IE exist in many texts, with the fundamental description given by (IMO, 2008). In brief, an IE is conducted by forcibly inclining the ship by moving a known weight a known transverse distance across the ship. The inclination is measured from the movement of a plumb-line relative to a mark-board, that is horizontal when the ship is upright. Typically, two or three plumb-lines are employed (forward-amidships-aft) to account for any torsional deformation of the ship. Then, the metacentric height $\overline{GM}$ is obtained according to,

$$\overline{GM} = \frac{wd}{\rho \nabla \tan\theta} \tag{1}$$

where w is the mass of the weight moved, d is the distance the weight is moved, ρ is the water density, ∇ the displaced volume of the ship and θ is the induced heel-angle. Eq. (2) calculates the height of the metacentre above the centre-of-buoyancy as a function-of-form for the given draught.

$$\overline{BM} = \frac{I}{\nabla} \tag{2}$$

In Eq. (2), I is the transverse second moment of area of the water-plane at that draught. The height of the centre-of-buoyancy above the keel $\overline{KB}$, (the centroid of volume at that draught) being a geometric property, is readily calculated from the hydrostatic particulars. The height of the mass-centroid (centre of gravity) above the keel $\overline{KG}$, is then given by Eq. (3).

$$\overline{KG} = \overline{KB} + \overline{BM} - \overline{GM} \tag{3}$$

2.2. Overview of experimental uncertainty analysis

The expression of experimental uncertainty is generally dealt with by National Metrology Institutions. However, for the application of specific procedures, scientific committees or societies more often take responsibility. Considering hydrodynamic testing, the International Towing Tank Conference (ITTC) provides Procedures and Guidelines for many aspects of ship related testing. Though the IE is not within its scope; one procedure (ITTC, 2008) does have relevant information, as it describes the application of uncertainty to hydrodynamic testing. Also, the development of all new procedures and guidelines should be expressed in line with the International Organisation for Standards (ISO), Guide to the Expression of Uncertainty in Measurement (ISO/IEC, 1995).

In accordance with ISO uncertainties can be categorised into Type-A and Type-B. Type-A uncertainties are components obtained utilising a method based on statistical analysis of a series of observations. Type-B uncertainties are components obtained by means other than repeated observations. For the IE most measurements are Type-B; or at least must be treated as such due to the nature of the measurement methods applied. In many respects however, the distinction is arbitrary as, for onward calculations, Type-A and Type-B uncertainties are treated in the same way. In its most simple form, the combined uncertainty in a result $u_c(y)$ is the root-sum-square of the standard uncertainties $u(x_i)$ for each ith input variable multiplied by a corresponding sensitivity coefficient c_i for each variable, given by Eq. (4).

$$u_c^2(y) = \sum_{i=1}^{N} c_i^2 u^2(x_i) \tag{4}$$

Of course, this is a somewhat simplified form, neglecting the possibility of correlation between various variables. Such correlation will be dealt with later in the paper, but for the immediate discussion this simplified form is sufficient. The sensitivity coefficient c_i is the partial derivative of the results with respect to any given input variable x_i; given by Eq. (5).

$$c_i = \frac{\partial y}{\partial x_i} \tag{5}$$

The standard uncertainty of any given variable is relatively easy to obtain. If a sufficiently large number of samples of measurement data are available, the Type-A standard uncertainty for a single sample is equal to the sample standard deviation. If there is no recent measurement data available, the limits of the uncertainty need to be estimated or e.g. taken from a specification of a measurement device. With these limits and an assumed probability distribution, the Type-B standard uncertainty can be derived (for application guidance see (ISO/IEC, 1995) Section 4.3).

3. Derivation of sensitivity coefficients

By assuming linearity, for small changes in draught T, for the variables $\overline{KB}$, I and ∇, the sensitivity coefficients can be obtained directly. Going to the hydrostatic tables for the ship, the tangent to the curves at the lightship 'as inclined' draught are utilised to obtain the coefficient α_n and constant terms β_n shown in Eq. (6).

$$\begin{aligned} \overline{KB} &= \alpha_1 T + \beta_1 \\ I &= \alpha_2 T + \beta_2 \\ \nabla &= \alpha_3 T + \beta_3 \end{aligned} \tag{6}$$

Eq. (7) is obtained by substituting Eqs. (1), (2) and (6) back into Eq. (3).

$$KG^{(\alpha_1 T+\beta_1)+}\left(\frac{\alpha_2 T+\beta_2}{\alpha_3 T+\beta_3}\right) - \left[\frac{wd}{\rho(\alpha_3 T+\beta_3)\tan\theta}\right] \tag{7}$$

Simplifying as much as possible, the relevant sensitivity coefficients are then given by Eqs. (8)–(12), for the ith heel-angle measurement induced by weight shift. In Eq. (12) the gradient terms α_n are replaced with the specific differential terms, as they are perhaps more meaningful.

$$c_{1i} = \frac{\partial \overline{KG}}{\partial \theta_i} = \frac{wd}{\rho \nabla \sin^2\theta_i} \tag{8}$$

$$c_{2i} = \frac{\partial \overline{KG}}{\partial \rho} = \frac{wd}{\rho^2 \nabla \tan\theta_i} \tag{9}$$

$$c_{3i} = \frac{\partial \overline{KG}}{\partial w} = -\frac{d}{\rho \nabla \tan\theta_i} \tag{10}$$

$$c_{4i} = \frac{\partial \overline{KG}}{\partial d} = -\frac{w}{\rho \nabla \tan\theta_i} \tag{11}$$

$$c_{5i} = \frac{\partial \overline{KG}}{\partial T} = \frac{\partial \overline{KB}}{\partial T} + \frac{1}{\nabla}\left(\frac{\partial I}{\partial T} - \frac{\partial \nabla}{\partial T}\overline{BM} + \frac{\partial \nabla}{\partial T}\frac{wd}{\rho \nabla \tan\theta_i}\right) \tag{12}$$

The uncertainty in the ship geometry is an important consideration in comparison to the drawings. This takes into account the uncertainty in the position of the centre-of-buoyance and the metacentre, from which all other calculations are taken. Taking the partial derivatives of Eq. (3) (with Eqs. (1) and (2) substituted accordingly) the sensitivity coefficients given by Eqs. (13)–(15) are obtained.

$$c_6 = \frac{\partial \overline{KG}}{\partial \nabla} = \frac{1}{\nabla^2}\left(\frac{wd}{\rho \tan\theta_i} - I\right) \tag{13}$$

$$c_7 = \frac{\partial \overline{KG}}{\partial I} = \frac{1}{\nabla} \tag{14}$$