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Computations of linear and nonlinear ship waves by higher-order boundary element method

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ABSTRACT

A practical method for computing ship waves accurately and efficiently is favorable for hull form optimization in early stage of ship design. In the present study, Rankine source method incorporated with high-order boundary element (HOBEM) discretization is applied to solve linear ship waves at first. Numerical implementation is described in detail. An incomplete LU factorization preconditioned Generalized Minimal Residual method (GMRES) is employed to solve the resulting boundary integral equation in order to improve efficiency. A corresponding Fortran program is developed for the validation study. It is applied to solve ship waves of different hulls, including slender Wigley hull, KRISO Container Ship (KCS) with relatively full form and a fishery patrol boat. Ship wave drag, sinkage, trim and wave pattern over a wide range of Froude numbers are all well predicted.

In order to further investigate nonlinear effects on ship waves, a fully nonlinear potential flow method based on stationary iteration is proposed. The same numerical approach of HOBEM is employed. The combined nonlinear free surface condition is solved in each iteration to evaluate the free surface. Numerical investigation for Wigley and KCS hull shows the present nonlinear method are accurate. Comparison study with linear and partial nonlinear solution are also carried out and nonlinear effects on ship waves are detailedly discussed.

1. Introduction

Solving ship wave problems accurately and efficiently is very important to evaluate ship resistance performance. The corresponding results like wave drag, sinkage and trim are regarded as the main criterions for hull form design and line optimization. Besides, steady flow also has influence on seakeeping performance of ship with forward speed and should be addressed in ship motion prediction (Kim and Shin, 2007). Compared with the experimental way, numerical prediction of high efficiency and accuracy at low cost is commonly chosen for practical ship design in early stage. The boundary element method (BEM) has played an important role in the ship wave problem for its virtue of transforming three dimensional problem into two dimensional one. As for BEM, most of computational amount lies in computation of Green's function and solving the resulting boundary integral equations. Robust schemes incorporated Green's function or solution of boundary integral equation are still needed to be sought.

To solve steady potential flow around an advancing ship, one can adopt Kelvin source Green's function, one kind of free surface Green's function, which satisfies Laplace equation, linearized free surface condition and radiation condition. The sources or dipoles need to be distributed on the surface of ship hull only. So the number of unknowns in boundary integral equations is relatively less than that which of Rankine source method. However, lots of CPU time would be consumed in the computation of Green's function. The Neumann–Kelvin (NK) theory proposed by Brard (1972) is a typical free surface Green's function method. Many studies show solutions by the method are not satisfactory for ship due to existence of waterline integral which is hard to compute. Noblesse et al. (2013) eliminated the term of waterline integral in NK theory by mathematical transformation and developed a new theory referred to as Neumann–Michell (NM) theory. According to NM theory, free surface Green's function is simplified and Hogner's approximation is taken into account as initial approximation, and then flow velocity potentials are solved by an iterative scheme. Ship wave drag, sinkage, trim and wave profiles predicted by NM theory are in overall good agreement with experiments in the reference (Huang et al., 2013).

Besides the above methodology, an alternative way is to use Rankine source method, of which the elementary solution $1/r$ is

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employed and distributed on all fluid boundaries. Mathematical formulation of Rankine source method is simple. And it is capable of being extended to nonlinear hydrodynamic problems. Dawson (1977) first applied Rankine source method to solve ship waves. Subsequently, lots of efforts have been done to improve accuracy, stability and efficiency of numerical approach. For instance, Raven (1988) did sufficient numerical investigations and proposed a few improvements of Dawson's method. Raven (1996) also developed a nonlinear Dawson's method based on stationary iterative procedure and each iteration solves a linearized problem with respect to the previous solution. Wyatt (2000) proposed an alternative nonlinear method by solving both kinematic and dynamic free surface condition in nonlinear iteration. Park et al. (2003) carried out comparative study of nonlinear Dawson's method and viscous flow method. Tarafder and Suzuki (2008) came up with a modified Rankine source method to predict ship wave drag. Peng et al. (2014) also used modified Dawson method in study of wave pattern and resistance of full form ship.

A common feature of the above study is that constant panels are adopted to discretize free surface and hull boundary. Flat elements usually poorly represent curved ship hull unless sufficient number of meshes are divided. Another weakness of constant panel method is that variables like velocity potential or source strength are discontinuous on boundary. By contrast, high order boundary element method (HOBEM) can construct continuous and high-order distribution of variables. So by using HOBEM, even with much fewer elements, higher accuracy can be reached than by using constant panel method. Gao (2006), Gao et al. (2008), Belibassakis et al. (2013) once applied non-uniform rational B-spline (NURBS) based BEM to steady ship wave problems.

The main purpose of our present work is to develop a practical and accurate numerical tool to compute ship waves. The HOBEM based on quadratic shape function with Rankine source distribution is adopted in this study. Linear ship wave problem in terms of uniform stream is considered at first and in view of it the numerical implementation is given in detail. By using Rankine source method, plenty of time is always consumed in solving matrix, the Generalized Minimal Residual algorithm (GMRES) designed for non-symmetric matrix (Saad and Schultz, 1986) is employed in the study in order to improve the calculation efficiency. As using raised panel approach on free surface leads to a non-diagonally dominant coefficient matrix, an incomplete LU factorization (ILU) (Saad, 1994) preconditioner is imposed on matrix before iteration. For validation and verification, a Fortran code is developed to compute ship waves for hull forms including slender Wigley, full form KRISO Container Ship (KCS) and a fishery patrol boat. Ship wave drag, sinkage, trim and wave pattern are predicted.

Results in comparison to experimental data indicate present method is generally accurate and robust. Further discussions show some nonlinear effects on ship waves should be addressed. A fully nonlinear potential flow method is then proposed. The method is based on stationary iteration with combined nonlinear free surface condition solved in each iteration. The same numerical approach of HOBEM is employed and performed by the developed Fortran code. Numerical investigations are carried out and comparison study of nonlinear method and linear solution for Wigley and KCS, and detailed discussions are done. Study shows that nonlinear effects on ship wave drag, sinkage and trim appear apparently at high Froude numbers and for free surface wave, influence of nonlinearity mainly exists in region near hull.

2. Mathematical model of ship waves

2.1. Governing equation

For ship wave problem, a ship moving forward with constant speed U is considered and a Cartesian coordinate $O\text{-}xyz$ which translates with ship is defined as shown in Fig. 1. xoy plane is on the undisturbed free surface with x axis forwards and z axis vertically upwards.

According to potential flow theory, fluid is assumed to be incompressible, inviscid and the flow is irrotational. Velocity potential satisfies Laplace equation in fluid domain.

$$\nabla^2\phi=0, \tag{1}$$

It is assumed total velocity potential ϕ is a sum of uniform stream potential $-Ux$ and disturbance potential φ,

$$\phi=-Ux+\varphi. \tag{2}$$

Boundary value problem about φ can be constructed by the following boundary conditions.

(1) Linearized free surface conditions is,

$$U^2\varphi_{xx}+g\varphi_z=0, \tag{3}$$

where subscripts xx and z mean partial derivatives.

(2) Hull boundary condition ensures fluid particles cannot penetrate ship hull,

$$\nabla(-Ux+\varphi)\cdot\mathbf{n}=0, \tag{4}$$

where $\mathbf{n}=(n_x,\ n_y,\ n_z)$ is normal vector of boundary surface and points outside of fluid domain.

(3) Radiation condition ensures no free surface waves propagate upstream.

$$\varphi=O\left(\frac{1}{\sqrt{x^2+y^2}}\right)\quad(x\to+\infty). \tag{5}$$

In numerical computation, free surface is truncated at finite distance and radiation condition is satisfied by using technique of shifting source points slightly downstream relative to collection points. As a result, self-influence of source and collection points can generate tangential velocities (Liu, 2003).

2.2. Calculation of wave drag, sinkage, trim and wave pattern

Once disturbance potential φ is solved, hydrodynamic pressure on hull can be computed by applying Bernoulli's equation,

$$\frac{p}{\rho}=U\varphi_x-\frac{1}{2}\nabla\varphi\cdot\nabla\varphi. \tag{6}$$

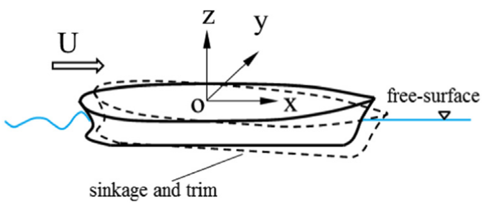

Fig. 1. Sketch of coordinate system.