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Water wave scattering by a finite dock over a step-type bottom topography

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ABSTRACT

The two-dimensional problem of water wave scattering by a dock of finite width present in water with a step-type bottom topography is investigated assuming linear theory. Two cases are considered. In the first case water wave is incident on the dock from the lower depth region and in the second case the wave is incident on the dock from the higher depth region. By applying Havelock's expansion formula for water wave potential along with the matching conditions, the boundary value problem in each case is reduced to a linear system of algebraic equations. The system of equations are then solved numerically after truncation and numerical estimates of the reflection and transmission coefficients and motion characteristics such as the force and the moment on the dock, for different values of various parameters and the wavenumber are obtained. These are depicted graphically against the wavenumber in a number of figures. Energy relation is also derived and used to check the accuracy of the computational results.

1. Introduction

Over the years, the problems of water wave propagation in the presence of floating bodies of various geometrical shapes are being studied due to their importance in ocean engineering. The interaction of water waves with a thin rigid plate floating on the free surface is important since it models a floating breakwater, a floating dock etc. The problem of water wave scattering by a semi-infinite rigid dock floating on infinitely deep water is of classical nature. It was first studied by Friedrichs and Lewy (1948) using the method of complex variable. Heins (1948) employed the Weiner–Hopf technique to study the problem for water of uniform finite depth. Chakrabarti et al. (2005) re-examined this problem utilizing a Fourier type of analysis, giving rise to Carleman type singular integral equations. Problems related to a finite dock was first investigated by Haskind (1942) who employed complex variable theory in the mathematical analysis. Subsequently, workers such as Rubin (1954), Sparenberg (1957), Holford (1964a,b), Leppington (1968), Dorfmann and Savvin (1998), Linton (2001), Hermans (2003) and others used various mathematical techniques solve the problem some for infinitely deep water and some for water of uniform finite depth. However, topographic variation of the bottom of water is very common in most of the problems modelling wave propagation in an ocean as an ocean is rarely of uniform finite depth throughout. Thus it is worthwhile to consider wave propagation problems in water with variable bottom topography. The problems of water wave scattering by small bottom undulations on the bed of an ocean have been considered by many workers from time to time such as Miles (1981), Davies (1982), Davies and Heathershaw (1984), Mandal and Basu (1990), Martha and Bora (2007) and others, who employed some sort of perturbation of the bottom condition exploiting the smallness of the bottom undulations and obtained the reflection coefficient up to first order. However, one may also consider a step-type bottom topography wherein an abrupt change in water depth occurs. Water wave propagation in the presence of changes in the depth of water is important in many coastal situations such as the passage of waves over a continental shelf as has been mentioned by Newman (1965). Reflection and transmission of water waves take place as the waves encounter the variable bottom topography. The problem of scattering of an incoming train of surface water waves by a sudden change of depth (finite to infinite) has been studied earlier by Sretenskii (1950) and Bartholomeusz (1958) using long wave approximation and by Newman (1965) by an integral equation procedure. Rhee (1997) obtained the first- and second-order solutions to the problem of wave transmission over a step using an integral equation with a finite depth Green's function in oblique waves and found the first-order transmission and reflection coefficients to be consistent with results obtained by Newman

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(1965) by using the method of matching of eigenfunction expansions. Karmakar and Sahoo (2008) investigated water wave scattering by a semi-infinite and a finite floating membrane in the presence of an abrupt change in bottom topography. Bhattacharjee and Soares (2010) studied water wave scattering by a floating structure near a wall with stepped bottom topography. The aforesaid problems were solved mostly by the eigenfunction expansion method along with the matching technique. The basic principle of the method is to apply the appropriate matching conditions along with the associated orthogonal relations of the eigenfunctions to reduce the boundary value problem into a linear system of algebraic equations. The system of equations is then solved to determine the unknown coefficients occurring in the eigenfunction expansion. It may be mentioned here that this matching technique has been introduced long back to study water wave problems involving structures of various geometrical in the linearized theory of water waves. For example, Takano (1960) employed a somewhat similar technique perhaps for the first time while studying the effect of surface water waves normally incident on an elevated sill and a fixed obstacle at the surface. Later, a modification of this technique has been employed by many researches on water waves. Garrett (1971) used this method to study water wave scattering by a circular dock. Sabuncu and Calisal (1981) used this method to compute hydrodynamics coefficients for vertical circular cylinders at finite water depth. Kirby and Dalrymple (1983) employed this method to study oblique wave scattering by an asymmetric trench. Hwang and Tang (1986) studied the effect of short waves on a fixed rectangular surface obstacle by using this method. More recently, Zheng et al. (2006) employed this technique in the study of wave radiation by a floating rectangular structure.

Recently, Dhillon and Banerjea (2015) and Dhillon et al. (2013) studied the problem of wave scattering by a semi-infinite rigid dock and a finite dock respectively in the presence of bottom undulations described by a sinusoidal and an exponentially decaying profile. In the present paper, the two-dimensional problem of water wave scattering by a rigid dock of finite width floating on water with a variable bottom having a step-type profile is analyzed. This may be regarded as an extension of the first-order problem considered by Rhee (1997) in the presence of a thin rigid dock of finite width floating on the free surface. Two cases are taken into consideration. In the first case the wave is incident on the dock from the region of lower depth water and in the second case the wave is incident on the dock from the region of higher depth water. In both the cases, the dock lies symmetrically with respect to the bottom step. Also the case when the dock does not lie symmetrically with respect to the bottom step has been considered for the situation when the wave is incident on the dock from the lower depth region. The other situation, namely when wave is incident on the dock from higher depth region, can also be analyzed similarly. The method of eigenfunction expansion followed by matching conditions is used to reduce the problem into a system of linear equations which are solved numerically after truncation, to obtain numerical estimates for the reflection and transmission coefficients and the motion characteristics such as the force and the moment on the dock. These coefficients are then depicted graphically against the wavenumber in a number of figures for different values of various parameters for both the cases. The motion characteristics are also depicted graphically against the wavenumber only for the case when the wave is incident on the dock from the region of lower depth water where the dock lies symmetrically with respect to the bottom step. For other cases similar figures for the motion characteristics can be obtained.

2. Waves incident from lower depth region

2.1. Dock situated symmetrically with respect to the bottom step

We choose a rectangular Cartesian co-ordinate system in which the $y-$axis is chosen vertically downwards into the fluid region and $xz-$plane is the rest position of the free surface (horizontal). $(x \le 0, 0 \le y \le h_1, -\infty < z < \infty)$ denotes the lower depth region while $(x \ge 0, 0 \le y \le h_2\ (>h_1), -\infty < z < \infty)$ denotes the higher depth region. The rigid floating dock lies symmetrically with respect to the bottom step so that its position is described by $(-a \le x \le a, y=0, -\infty < z < \infty)$ (see Fig. i). The domain of consideration is divided into four regions, region 1 $\{-\infty < x < -a, y=h_1\}$, region 2 $\{-a < x < 0, y=h_1\}$, region 3 $\{0 < x < a, y=h_2\}$, region 4 $\{a < x < \infty, y=h_2\}$, as shown in Fig. i. Assuming linear theory and the motion in water to be irrotational, time-harmonic and independent of the co-ordinate z, the motion in region j can be described by the velocity potential $Re\{(ga/\omega)\phi_j(x,y)e^{-i\omega t}\}$ $(j=1,2,3,4)$ whereω is the angular frequency, g is the acceleration due to gravity. Then $\phi_j(x,y)$ $(j=1,2,3,4)$ satisfies

$$\nabla^2\phi_j = 0 \quad \text{in region } j, \tag{2.1}$$

In region 1, ϕ_1 satisfies the free surface condition

$$K\phi_1 + \phi_{1y} = 0 \quad \text{on } y=0,\ -\infty < x < -a \tag{2.2}$$

where $K=\omega^2/g$, along with the bottom condition

$$\phi_{1y} = 0 \quad \text{on } y=h_1,\ -\infty < x < -a \tag{2.3}$$

In region 2, ϕ_2 satisfies the conditions

$$\phi_{2y} = 0 \quad \text{on } y=0,\ -a < x < 0 \tag{2.4}$$

just below the rigid dock and

$$\phi_{2y} = 0 \quad \text{on } y=h_1,\ -a < x < 0 \tag{2.5}$$

on the bottom.

In region 3, ϕ_3 satisfies

$$\phi_{3y} = 0 \quad \text{on } y=0,\ 0 < x < a, \tag{2.6}$$

$$\phi_{3y} = 0 \quad \text{on } y=h_2,\ 0 < x < a, \tag{2.7}$$

$$\phi_{3x} = 0 \quad \text{on } x=0,\ h_1 < y < h_2. \tag{2.8}$$

In region 4, ϕ_4 satisfies the free surface condition

$$K\phi_4 + \phi_{4y} = 0 \quad \text{on } y=0,\ a < x < \infty \tag{2.9}$$

along with the bottom condition

$$\phi_{4y} = 0 \quad \text{on } y=h_2,\ a < x < \infty. \tag{2.10}$$

One final condition which specifies the nature of solution near the dock edges, $(\pm a, 0)$, has ben derived by Linton (2001) and is given by

$$\frac{\partial\phi_j}{\partial r} \sim A \ln r \quad \text{as } r \to 0 \tag{2.11}$$

for some constant A, where $r^2 = [(x+a)^2 + y^2]$ for $j=1,2$ and $r^2 = [(x-a)^2 + y^2]$ for $j=3,4$.

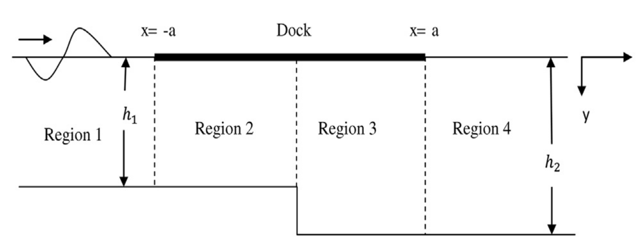

Fig. i. Schematic diagram.