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Validation of Boussinesq–Green–Naghdi modeling for surf zone hydrodynamics

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ABSTRACT

2D modeling for surf zone phenomenons are validated in present work using the rotational Boussinesq–Green–Naghdi model. Three benchmark test cases are simulated: tsunami wave runup a conical island; tsunami wave runup complex shelf; and rip current and wave setup over sand bars. The computed results are compared to the experimental data including the free surface deformation and depth-averaged velocities. The simulated 2D cases fundamentally validate the model's ability in predicting wave transformation, wave breaking, wave runup and the velocity field for complex hydrodynamic conditions and give the basis for moving on to more complex applications. The lack of irrotationality would strongly contribute to the depth-varying velocity profile of rotational modeling, which has been partially proven in 1D undertow test (Zhang et al., 2014a). Unfortunately, few 2D experiments with rotational vortex data measured could be found due to the difficulty of recording the vortex characteristics. Future work would be the model application to much more complex geophysical and engineering problems, where the lack of any irrotational constrain is expected to excel.

1. Introduction

Boussinesq-type models describe nonlinear and dispersive wave propagation in arbitrary directions with good computational efficiency and range of validity. Since the classic depth-averaged Boussinesq theory was introduced by Peregrine et al. (1967), Boussinesq-type models have experienced a booming development in accuracy, range of application and popularity (Nwogu, 1993; Wei et al., 1995; Kennedy et al., 2001; Madsen and Schäffer, 1998; Gobbi and Kirby, 1999; Gobbi et al., 2000; Kennedy and Kirby, 2002; Lynett and Liu, 2002a; Madsen et al., 2002; Donahue et al., 2015). Properties such as dispersion, shoaling gradient, and higher order harmonics have been significantly improved by asymptotic rearrangement throughout the system. Introducing an eddy viscosity model and moving shorelines enables viscid Boussinesq models to describe surf zone dynamics such as wave evolution, wave setup, and large scale wave-induced currents including rip currents and longshore currents (Schäffer and Madsen, 1993; Sørensen et al., 1998; Chen et al., 2000; Kennedy et al., 2000a, 2000b; Lynett et al., 2002b; Musumeci et al., 2005; Nwogu and Demirbilek, 2010; Shi et al., 2012; Bonneton et al., 2011; Kim and Lynett, 2013).

Green and Naghdi (1976) introduced an alternate but related approach dealing with the rotational orbital velocities using a polynomial structure of the velocity profile. The mass and momentum equations were solved using a weighted residual with no irrotationality constraint, so that the rotational flow is expected to be modeled naturally. Zhang et al. (2012, 2013) developed Boussinesq–Green–Naghdi models that take advantage of both Boussinesq and Green–Naghdi systems. Polynomial expansions (Shields and Webster, 1988) and Boussinesq scaling were both applied. The major interest, rotational modeling, has been partially validated at both $O(\mu^2)$ and $O(\mu^4)$ approximation levels for 1D test cases which include nonbreaking and breaking waves in surf zone, undertow and wave runup (Zhang et al., 2014b; Panda et al., 2014). All these give the basis and confidence for further 2D surf zone modeling.

For breaking techniques, two common ones are usually adopted. The first technique is the surface roller method based on the flux version of Boussinesq equations (Schäffer and Madsen, 1993) which was further developed by Madsen et al. (1997). The second approach is the ad hoc eddy viscosity formulation originally

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developed by Zelt (1991), and extended by Kennedy et al. (2000a) that yields extra viscous terms in momentum equations leading to wave dissipation. Both eddy viscosity and surface roller techniques are of comparable accuracy. A third, more recent, approach turns off dispersive terms in the vicinity of the breaking roller and allows the dissipative nature of shallow water bores to remove energy from waves while conserving momentum (Shi et al., 2012; Tissier et al., 2012).

In this paper, the rotational Boussinesq–Green–Naghdi modeling is extended to 2D/quasi-3D demonstrations for complex hydrodynamics, including wave transformations and interactions, wave breaking, and moving multi-wet–dry shoreline interfaces. In order to reproduce the energy dissipation under the breaking wave crest, viscous terms in the Navier Stokes equation represented by eddy viscosity are kept by Boussinesq scaling which may be further improved by introducing extra scaling. The eddy viscosity is modeled by the depth-integrated $k-l$ turbulent-kinetic-energy equation. The $k-l$ model (Karambas and Koutitas, 1992) is a basic approach for turbulence modeling compared to more complex turbulence models such as $k-\epsilon$ model, $k-\omega$ model, and Reynolds-stress related models. The model is arguably the simplest incomplete turbulence model, and hence it has the broad range of applicability with much less computational cost. This simplicity is helpful to make the system tractable. And similar to $k-\epsilon$ model (King et al., 2012), $k-\omega$ (Neary, 2003) model, $k-l$ is a favorable numerical coupling between the flow and turbulence equations. Benchmarked experimental cases with wave breaking, wave runup and rotational velocity fields are simulated. Numerical results reach highly satisfactory level compared to collected experimental data.

Due to the single surface assumption in Boussinesq models, there is an upper limit on accuracy for the model used here to simulate the complex free surfaces under wave breaking, in the roller region and near the wave crest. Still, the present model maintains a balance between accuracy and computational efficiency, which is ideal for intermediate-scale modeling with much lower computational cost than full Navier–Stokes solvers (e.g. Ma et al., 2012; Higuera et al., 2013).

2. Boussinesq–Green–Naghdi rotational water wave system

2.1. Scaling

The system derived is dimensionless, applying Boussinesq-shallow water scaling for non-dimensional variables, which are defined aswhere the superscript $*$ indicates dimensional variables. $\mathbf{x}^* \equiv (x^*, y^*)$ are horizontal spatial coordinates and the vertical coordinate (z^*) is oriented positive upward. g^* is gravitational acceleration. Time t^* is scaled by long wave celerity $(g_0 h_0)^{1/2}$ and wavenumber k_0, while depth h^* and surface elevation η^* scales with typical water depth h_0. Horizontal and vertical velocities (u^*, v^*, w^*) are all scaled by wave orbital velocities. The pressure P^* is hydrostatically scaled. This scaling allows strongly nonlinear waves, although of course the system also remains valid for small amplitude waves. Eddy viscosity ν_t^* is assumed to scale with depth and gravity, and turbulent stresses use both eddy viscosity scaling and Boussinesq-shallow water scaling.

$$\begin{aligned}
&(x,y)=k_0(x^*,y^*), \quad z=h_0^{-1}z^*, \quad t=k_0(g_0h_0)^{1/2}t^*, \quad h=h_0^{-1}h^*, \\
&\eta=(h_0)^{-1}\eta^*, \quad P=(\rho^* g_0 h_0)^{-1}P^*, \quad g=g_0^{-1}g^*, \quad (u,v)=(g_0h_0)^{-1/2}(u^*,v^*), \\
&w=(k_0h_0)^{-1}(g_0h_0)^{-1/2}w^*, \quad \nu_t=h_0^{-1}(g_0h_0)^{-1/2}(k_0h_0)^{-1}\nu_t^*, \quad \tau_{\mathbf{xx}}=g_0^{-1}k_0^{-1}h_0^{-2}\tau_{\mathbf{xx}}^*, \quad \tau_{z\mathbf{x}}=g_0^{-1}k_0^{-2}h_0^{-3}\tau_{z\mathbf{x}}^*
\end{aligned} \tag{2.1}$$

2.2. Model equations

As derived by Zhang et al. (2013), the fully $O(\mu^2)$ Boussinesq–Green–Naghdi rotational water wave model assumes a polynomial expansion for the horizontal and vertical velocity,

$$\begin{aligned}
&\mathbf{u}=\mathbf{u}_0+\mu^2\mathbf{u}_1 f_1+\mu^2\mathbf{u}_2 f_2+O(\mu^4) \\
&w=-\nabla\cdot\mathbf{u}_0(\eta+h)q-\mathbf{u}_0\cdot\nabla h+O(\mu^2)
\end{aligned} \tag{2.2}$$

where μ is a dimensionless indicator of approximation level; $\mathbf{u}=(u,v)$; $q=(z+h)/(h+\eta)$. The polynomial basis functions $f_n(q)$ used here are,

$$\begin{aligned}
&f_0=1 \\
&f_1=-54/125+q \\
&f_2=-1/5+q^2,
\end{aligned} \tag{2.3}$$

which is the optimized set at the approximation level of $O(\mu^2)$. All horizontal velocity components $\mathbf{u}_0, \mathbf{u}_1, \mathbf{u}_2$ are independent, so that higher order velocity components do not depend on lower order components as in other irrotational Boussinesq theories (e.g. Peregrine et al., 1967).

Insertion of the velocity expressions (2.2) into the vertically integrated mass and momentum equations gives, at $O(\mu^2)$,

$$\eta_{,t}+\nabla\cdot\left(\mathbf{u}_0(\eta+h)+\mu^2\sum_{n=1}^{2}\mathbf{u}_n(\eta+h)g_n|_{q=1}\right)=0 \tag{2.4}$$

$$\begin{aligned}
&\mathbf{u}_{0,t}(\eta+h)g_m|_{q=1}+\mathbf{u}_0\cdot\nabla\mathbf{u}_0(\eta+h)g_m|_{q=1}+g\nabla\eta(\eta+h)g_m|_{q=1} \\
&+\mu^2\sum_{n=1}^{2}\left(\mathbf{u}_{n,t}(\eta+h)\phi_{mn}-\mathbf{u}_n\eta_{,t}\varepsilon_{mn}\right)|_{q=1} \\
&-\mu^2\Big[\frac{1}{2}\nabla(\nabla\cdot\mathbf{u}_{0,t})(\eta+h)^3(g_m-\nu_m)+(\nabla\cdot\mathbf{u}_{0,t})(\eta+h)^2\nabla(\eta+h)g_m \\
&+\nabla(\mathbf{u}_{0,t}\cdot\nabla h)(\eta+h)^2(g_m-S_m)+\mathbf{u}_{0,t}\cdot\nabla h\nabla\eta(\eta+h)g_m \\
&-(\nabla\cdot\mathbf{u}_{0,t})(\eta+h)^2\nabla hS_m\Big]|_{q=1} \\
&+\mu^2\sum_{n=1}^{2}\big[(\mathbf{u}_n\cdot\nabla\mathbf{u}_0+\mathbf{u}_0\cdot\nabla\mathbf{u}_n)(\eta+h)\phi_{mn} \\
&-\mathbf{u}_n\nabla\cdot(\mathbf{u}_0(\eta+h))\varepsilon_{mn}\big]|_{q=1}+\mu^2(\eta+h)^2[(\nabla\cdot\mathbf{u}_0)^2 \\
&-\mathbf{u}_0\cdot\nabla(\nabla\cdot\mathbf{u}_0)](\nabla\eta g_m+\nabla h(g_m-S_m))|_{q=1}+\frac{\mu^2}{2}(\eta+h)^3\nabla[(\nabla\cdot\mathbf{u}_0)^2 \\
&-\mathbf{u}_0\cdot\nabla(\nabla\cdot\mathbf{u}_0)](g_m-\nu_m)|_{q=1} \\
&-\mu^2(\eta+h)\nabla\eta\mathbf{u}_0\cdot\nabla(\mathbf{u}_0\cdot\nabla h)g_m|_{q=1} \\
&-\mu^2(\eta+h)^2\nabla(\mathbf{u}_0\cdot\nabla(\mathbf{u}_0\cdot\nabla h))(g_m-S_m)|_{q=1}=\int_{-h}^{\eta}f_m\frac{\partial\tau_{\mathbf{x}z}}{\partial z}dz \\
&+\int_{-h}^{\eta}\mu^2 f_m\nabla\cdot\tau_{\mathbf{xx}}\,dz, \quad m=0,1,2
\end{aligned} \tag{2.5}$$

where g_m and S_m are integral functions of f_n, e.g. $g_n\equiv\int_0^q f_n(q)dq$, with many other functions defined (Appendix A).