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# An enhanced stiffness model for elastic lines and its application to the analysis of a moored floating offshore wind turbine

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ABSTRACT

The performance of a polyester mooring line is non-linear and its elongation plays a significant role in the dynamic response of an offshore moored structure. However, unlike chain, the tension–elongation relationship and the overall behavior of elastic polyester ropes are complex. In this paper, by applying an enhanced stiffness model of the mooring line, the traditional elastic rod theory has been extended to allow for large elongations. One beneficial feature of the present method is that the tangent stiffness matrix is symmetric; in non-linear formulations the tangent stiffness matrix is often non-symmetric. The static problem was solved by Newton–Raphson iteration, whereas a direct integration method was used for the dynamic problem. The computed mooring line tension was validated against the proprietary OrcaFlex software. Results of mooring line top tension predicated by different elongations are compared and discussed. The present method was then used for a simulation of an offshore floating wind turbine moored with taut lines. From a comparison between linear and non-linear formulations, it is seen that a linear spring model under-estimates the mean position when the turbine is operating, but over-estimates the amplitude of the platform response at low frequencies when the turbine has shut down.


## 1. Introduction

The capture of offshore wind energy plays a key role across the maritime industry (EWEA, 2013). Offshore wind turbines are becoming larger and more powerful, and are being deployed in ever-deeper water. They can be mounted on a fixed or floating base, but the former starts to lose its economic advantage for water depths larger than 60 m (Goupee et al., 2014). Although the mooring system design for a floating offshore wind turbine (FOWT) has benefited from the experience of offshore oil and gas platforms, there are still several unknowns dependent on the type of floating bodies, e.g. size and environmental loading. From a report of EWEA (2013), it is recommended that more research must be done on mooring and anchoring systems for wind turbines.

Numerical simulations of the dynamic response of mooring lines have been studied during the past few decades, for both elastic and inelastic lines. A massless spring (e.g. Kim et al., 2001) or the catenary equation (e.g. Agarwal and Jain, 2003) provide straightforward ways to model a mooring line, but it is difficult to account for the dynamic response and the interaction between the floating body and mooring line in an accurate manner. Multi-body system dynamics (e.g. Kreuzer and Wilke, 2003) divides the mooring line into several rigid bodies, but results in a large number of degrees of freedom even for a single line. Non-linear finite element methods (FEMs), accounting for geometric and material non-linearities, have been widely used for modelling mooring line response (e.g. Kim et al., 2013). Geometric non-linearity is needed for large displacements of the mooring line, while material non-linearity can model the time-dependent properties of a polyester rope, e.g. Young's modulus. However, a major disadvantage of FEM is the transformation between local coordinate and global coordinate, which is often computationally intensive. The lumped mass and spring method can be categorized as a non-linear FEM method, for which the shape function becomes a single line (Low, 2006).

Unlike traditional non-linear FEMs, the elastic rod theory is a global-coordinate-based method, which is considered to be more efficient (Kim et.al, 1994). The transformation between local and global coordinate is dealt within the element stiffness matrix. Following the elastic rod theory of Love (1944), Nordgren (1974) and Garrett (1982) developed this method and solved the governing non-linear equations by a finite difference method (FDM) and by FEM, respectively. Many researchers have further developed the elastic rod theory, including elongation of the line, seabed friction, non-linear material properties and the incorporation of buoys or clump weights in the mooring line model. Paulling and Webster (1986) considered the analysis of large amplitude

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motions of a TLP under the action of wind, wave and current, under the assumption of small line elongation. Ran (2000) proposed a finite element formulation for mooring lines and risers based on Garrett's rod theory, applicable to both frequency and time domains. Based on the traditional small extensible rod theory, the incorporation of large elongation has been presented by many researchers (e.g. Chen, 2002; Tahar, 2001; Kim et al., 2011).

Based on the successful experience from offshore oil & gas platforms, the design and modelling of a FOWT has tended to use the same mathematical modelling and methods of solution as for offshore platforms, e.g. the hydrodynamic analysis of floating body, mooring design and the types of FOWTs (Spar, TLP and Semisubmersible, etc). The design of a station-keeping system for a FOWT uses the same methods and guidance as for a floating platform (e.g. ABS, 2014a). However, the geometry and operational water depth are different. Also, the turbine thrust force may have an effect on the motion response of the floating body and mooring line tension, and vice versa. These differences need to be examined for a FOWT.

Polyester lines are made from visco-elastic materials and the stiffness characteristics rely on the loading history, the load duration and magnitude, etc (ABS, 2014b). The material nonlinearity of polyester lines is difficult to model and requires a longer simulation time. Thus some approximate models are used. For example, the dual stiffness method (Tahar et al., 2012) recommended by API RP 2SM (2001), and the visco-elastic model (Kim et al., 2011). In the present paper, a sensible balance has been sought between efficiency and accuracy. The traditional rod theory has been extended to allow for large stretch by applying an enhanced stiffness method. By using an approximation of the nonlinear tension-elongation relationship in a Taylor series expansion (Ćatipović et al., 2011), the mathematical and numerical formulation of large extensible mooring line are considered.

## 2. Mathematical formulation of a mooring line with large elongation

### 2.1. Equation of motion

For polyester mooring lines bending and torsion stiffness can be neglected, but the elongation cannot be assumed to be small. The mooring line is discretized into a number of rods and the centreline of each rod is described by a space-time curve $r(s,t)$. From Ćatipović et al. (2011), the equation of motion for a rod with large elongation can be written as:

$$\frac{d}{ds}\left(\frac{T_E}{1+\varepsilon}\frac{d\mathbf{r}}{ds}\right)+(1+\varepsilon)\mathbf{q}_E=(1+\varepsilon)\rho\ddot{\mathbf{r}} \tag{1}$$

where $\tilde{m}$ and $\tilde{A}$ are distributed mass and cross-section area after extension.$\rho$ and $\mathbf{g}$ are sea water density and gravitational acceleration, respectively.$\ddot{\mathbf{r}}$ represents the time derivation of the rod. $T_E$ is the effective tension of the rod. The relation between the real tension $T_R$ and the effective tension are (Ćatipović et al., 2011): $T_E=T_R+pA$, where $p$ and $A$ are hydrostatic pressure and cross-section area, respectively. $\mathbf{q}_E$ is the load acting on the rod. For static problem,$\mathbf{q}_E=\tilde{m}\mathbf{g}-\rho\tilde{A}\mathbf{g}$, while for dynamic problem $\mathbf{q}_E=\tilde{m}\mathbf{g}-\rho\tilde{A}\mathbf{g}+\mathbf{F}_H$.

in which $\mathbf{F}_H$ is the hydrodynamic loads on the mooring line (Paulling and Webster, 1986) calculated by Morison's equation (1950) as

$$\mathbf{F}_H=-C_A\rho A\ddot{\mathbf{r}}_n+C_M\rho A\dot{\mathbf{V}}_n+\frac{1}{2}C_D\rho D\left|\mathbf{V}_n-\dot{\mathbf{r}}_n\right|(\mathbf{V}_n-\dot{\mathbf{r}}_n) \tag{2}$$

where $n$ denotes the normal component. $C_A$,$C_M$ and $C_D$ are the added mass, inertial (Morison) and drag coefficients.

Mooring line normal component of acceleration $\ddot{\mathbf{r}}_n$, normal component of velocity $\dot{\mathbf{r}}_n$, normal component of water particle velocity $\mathbf{V}_n$ and water practical acceleration $\dot{\mathbf{V}}_n$ are given by (Ćatipović et al., 2011)

$$\dot{\mathbf{r}}_n=\dot{\mathbf{r}}-\left(\dot{\mathbf{r}}\cdot\frac{d\mathbf{r}}{ds}\right)\frac{d\mathbf{r}}{ds},\ddot{\mathbf{r}}_n=\ddot{\mathbf{r}}-\left(\ddot{\mathbf{r}}\cdot\frac{d\mathbf{r}}{ds}\right)\frac{d\mathbf{r}}{ds} \tag{3}$$

$$\dot{\mathbf{V}}_n=\dot{\mathbf{V}}-\left(\dot{\mathbf{V}}\cdot\frac{d\mathbf{r}}{ds}\right)\frac{d\mathbf{r}}{ds},\ddot{\mathbf{V}}_n=\ddot{\mathbf{V}}-\left(\ddot{\mathbf{V}}\cdot\frac{d\mathbf{r}}{ds}\right)\frac{d\mathbf{r}}{ds} \tag{4}$$

In Eq. (1), $\varepsilon$ is the elongation of the rod. Following Ćatipović et al., assuming equal principal stiffness, the relationship between the effective tension and elongation can be written as

$$\varepsilon=\frac{T_E}{AE} \tag{5}$$

where $AE$ is the axial stiffness

The following elongation condition then has to be satisfied (Ćatipović et al., 2011)

$$\frac{1}{(1+\varepsilon)^2}\frac{d\mathbf{r}}{ds}\cdot\frac{d\mathbf{r}}{ds}=1 \tag{6}$$

In the static problem, the mass per unit length and diameter of the mooring line are related to the elongation $\varepsilon$. The cross-sectional area and mass after elongation can be written as $A/(1+\varepsilon)$ and $m/(1+\varepsilon)$, respectively (Ćatipović et al., 2011), where $A$ and $m$ are the cross-section area and mass of the mooring line without stretch. Applying the above relationship to the motion equation, we see that the term $(1+\varepsilon)$, multiplied by the applied force $\mathbf{q}_E$ cancels out. For the hydrodynamic force calculated by Morison's equation, the mass per unit length and cross-sectional area for one element were assumed constant.

Eqs. (1) and (6) show the rod motion equation and elongation condition, respectively: they are non-linear. In the following section, we will describe a numerical procedure for solving this nonlinear equation and the required order of approximation for the elongation condition.

### 2.2. Numerical Implementation

#### 2.2.1. Static problem

For the static problem, $\mathbf{r}$ is independent of time. Consequently the inertial term in Eq. (1) is deleted. We therefore have

$$\frac{d}{ds}\left(\frac{T_E}{1+\varepsilon}\frac{d\mathbf{r}}{ds}\right)+\mathbf{q}_E=0 \tag{7}$$

Using the Taylor series expansion, the elongation relationship can be written as:

$$\frac{1}{(1+\varepsilon)^2}=1-2\varepsilon+3\varepsilon^2+o(\varepsilon^3) \tag{8}$$

However, it is not clear, *a priori*, whether the third-order term should be included explicitly. In the present paper, the order of expansion and subsequent results will be discussed.

In the FEM, the variables $r_i$ and $T_E$ may be approximated (Garrett, 1982) as

$$r_i(s)=\sum_{k=1}^{4}A_k(s)U_{ik} \tag{9}$$

$$T_E(s)=\sum_{m=1}^{3}P_m(s)\lambda_m \tag{10}$$

where $A_k$ and $P_m$ are shape functions. The definition of the shape functions can be found in the Appendix A. $U_{ik}$ and $\lambda_m$ are unknown variables. The subscript $i$ of $U_{ik}$ denotes the dimension of the element. For the 3-dimensional problem, $i$=3. For $k$=1 and 3, $U_{ik}$