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Effect of compressibility on supercavitating flow around slender conical body moving at subsonic and supersonic speed

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ABSTRACT

Based on slender body theory (SBT), Tait's state equation and Riabouchinsky closure scheme, this paper has established a theoretical model and computational method including the effect of compressibility for supercavitating flow past a high speed slender conical body, derived the integer-differential equations (IDE) according to different characteristics of subsonic and supersonic flows, and presented the numerical discrete scheme and iteration method to solve the IDE. Supercavity shapes and the hydrodynamic coefficients are acquired for a slender conical body at different cone semi-angles, cavitation number and Mach number. The theoretical model and calculated results are verified by the comparison with the results of other literatures. Finally we have analyzed the influences of fluid compressibility on the supercavity shapes, pressure distribution over the cone surface and base drag coefficient. The above results show that the predicted accuracy of the supercavity shape, maximal radius, aspect ratio, drag coefficient is very good for small cone semi-angle till 15° both in incompressible and compressible flow.

1. Introduction

Supercavitation can reduce the friction drag of a moving underwater vehicle significantly. The high-speed supercavitating projectiles with remaining kinetic energy in the end of trajectory can be used to intercept torpedoes, destroy mines and break underwater obstacles. Vlasenko (2003), Savchenko (1997) and Kirschner (2001) have carried out some experiments on supercavitating projectiles, whose speeds were up to 1300, 1350 and 1549 m/s respectively which exceeded underwater sonic speed (1450 m/s). At present, supercavitating projectile has been further developed and greater progress has been made (Serebryakov, 2006, Serebryakov and Schnerr, 2003, Serebryakov et al., 2009).

Chou (1974), Guzevsky (1979), Kulkarni and Pratap (2000), Ohtani et al. (2006) have studied the characteristics of the supercavitating flow and projectile movement without taking fluid compressibility into account. Serebryakov (1973, 1992, 1994, 1998, 2001) have made theoretical studies of the incompressible and compressible supercavitating flow based on the SBT and Matched Asymptotic Expansions Method (MAEM). In this paper, we focus on a high speed supercavitating flow around slender conical body and take the fluid compressibility into account; a unified theoretical model and numerical method have been established for subsonic and supersonic supercavitating flow, and the derived nonlinear integer-differential equations are successfully solved by the proposed numerical discrete scheme and iteration method. These results will provide a useful basis for next research on the trajectory of high-speed supercavitating projectile.

2. Mathematical problems

A cylindrical coordinate system (x, r) need to be established at a slender cone base with x along the cone-cavity axis as shown in Fig. 1. The radius $r = r_1(x) = \varepsilon(x+l)$ of the cone is given in advance; the radius $r = R(x)$ and length L of the supercavity are to be computed. l and R_n are the length and base radius of the cone respectively, and $\varepsilon = R_n/l$ is a small parameter. Assuming the ideal compressible flow around the cone is irrotational and steady, the free stream velocity is U_∞. We adopt Riabouchinsky closure scheme at the end of supercavitation for subsonic flow, which is unnecessary for supersonic flow.

We suppose that the perturbation velocity potential is φ, the perturbation velocity is $\nabla\varphi$ and the fluid velocity is $(U_\infty \boldsymbol{i} + \nabla\varphi)$. The mathematical problems describing the subsonic and supersonic flow are as following:

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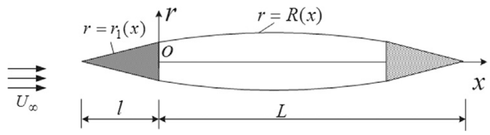

Fig. 1. Slender cone and coordinate system.

In the flow field, for $Ma_\infty<1$ or $Ma_\infty>1$, the governing equation is

$$(1-Ma_\infty^2)\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial r^2}+\frac{\partial\varphi}{r\partial r}=0 \tag{1}$$

where $Ma_\infty=U_\infty/a_\infty$, here Ma_∞ and a_∞ are the Mach number and the sonic speed of free stream at infinity respectively.

On the boundary of $r=r_1(x)$ or $r=R(x)$, the kinematic boundary condition is

$$\frac{\partial\varphi}{\partial r}=\left(U_\infty+\frac{\partial\varphi}{\partial x}\right)\frac{\mathrm{d}r}{\mathrm{d}x} \tag{2}$$

For $x,r\to\infty$, the perturbation attenuation condition is

$$\nabla\varphi\to 0 \tag{3}$$

At the flow separation point of cone base, there are at least the flow continuity condition and the energy conservation condition (Serebryakov, 2006, Serebryakov and Schnerr, 2003, Serebryakov et al., 2009). Here we adopt the flow continuity condition and it is assumed the cavity starts at $x=0$, and then at this point there is

$$r_1=R \text{ and } \frac{\mathrm{d}r_1}{\mathrm{d}x}=\frac{\mathrm{d}R}{\mathrm{d}x} \tag{4}$$

For water, the relationship between the pressure and density is expressed by the Tait's state equation, that is

$$\frac{p+B}{p_\infty+B}=\left(\frac{\rho}{\rho_\infty}\right)^n \tag{5}$$

where p_∞ and ρ_∞ are the pressure and density of free stream at infinity respectively; p and ρ are the pressure and density at some point in the flow field respectively; n is the ratio of specific heats, $n=7.15$ for water; B is a pressure constant, $B=2.98\times10^8$Pa.

The Bernoulli's equation is

$$\frac{n}{n-1}\frac{p+B}{\rho}+\frac{U^2}{2}=\frac{n}{n-1}\frac{p_\infty+B}{\rho_\infty}+\frac{U_\infty^2}{2} \tag{6}$$

For the isentropic flow, we have

$$a^2=\frac{\mathrm{d}p}{\mathrm{d}\rho}=n\frac{p+B}{\rho},\ a_\infty^2=n\frac{p_\infty+B}{\rho_\infty} \tag{7}$$

where a is the sonic speed at some point in the flow field.

According to Eqs. (5)–(7), for the flow around a slender body the pressure coefficient is derived as follows:

$$C_p=\frac{p-p_\infty}{0.5\rho_\infty U_\infty^2}=\frac{2}{nMa_\infty^2}\left\{\left[1-\frac{n-1}{2}Ma_\infty^2\left(\frac{2\varphi_x}{U_\infty}+\frac{\varphi_r^2}{U_\infty^2}\right)\right]^{\frac{n}{n-1}}-1\right\} \tag{8}$$

The natural cavitation number is defined as $\sigma=(p_\infty-p_v)/(0.5\rho_\infty U_\infty^2)$, here $p_\infty=p_a+\rho_\infty gh$, p_a is the local atmospheric pressure, ρ_∞ is the water density, g is the gravity acceleration, p_v is the saturated vapor pressure of water, and h is the height from the water surface to the center of the cone base. On the supercavity boundary $0\le x\le L-l$, $p=p_v$, so we have $C_p=-\sigma$.

3. Integro-differential equations

For $Ma_\infty<1$, Eq. (1) is an elliptic equation, which means that the disturbance will spread in the flow field around the disturbance source. For $Ma_\infty>1$, Eq. (1) is a hyperbolic equation, which means that the disturbance will only spread within the Mach cone downstream. According to Eqs. (1) and (3), the perturbation velocity potentials in the subsonic and supersonic flow can be written as follows respectively:

$$\varphi(x,r)=-\int_{-l}^{L}\frac{q(\xi)d\xi}{4\pi\sqrt{(x-\xi)^2+(mr)^2}},\ Ma_\infty<1 \tag{9}$$

$$\varphi(x,r)=-\int_{-l}^{x-mr}\frac{q(\xi)\mathrm{d}\xi}{2\pi\sqrt{(x-\xi)^2-(mr)^2}},\ Ma_\infty>1 \tag{10}$$

where $m=\sqrt{|1-Ma_\infty^2|}$, $q(\xi)=U_\infty\frac{\mathrm{d}S(x)}{\mathrm{d}x}|_{x=\xi}$, $S(x)=\pi r^2$, here $q(\xi)$ is the source strength distributed along the longitudinal axis at location ξ, and $S(x)$ represents the cross-sectional area of the body-cavity.

It is convenient to define a variable ζ as $\zeta=R^2$. Using Eqs. (2) and (4), and substituting Eqs. (9) and (10) into Eq. (8), after some tedious manipulation, we can obtain the nonlinear integro-differential equations describing the supercavity profiles ($0\le x\le L-l$) around the slender cone in subsonic and supersonic flow respectively:

$$\int_0^{L-l}\frac{d^2\zeta}{dx^2}|_{x=\xi}\frac{d\xi}{\sqrt{(x-\xi)^2+m^2\zeta}}=-2\sigma_m+\frac{1}{2\zeta}\left(\frac{\mathrm{d}\zeta}{\mathrm{d}x}\right)^2 -2\varepsilon^2\ln\frac{[x+l+\sqrt{(x+l)^2+m^2\zeta}][x-L+l+\sqrt{(x-L+l)^2+m^2\zeta}]}{[x+\sqrt{x^2+m^2\zeta}][x-L+\sqrt{(x-L)^2+m^2\zeta}]}\quad \text{for } Ma_\infty<1 \tag{11}$$

$$\int_0^{x-mR}\frac{\mathrm{d}^2\zeta}{\mathrm{d}x^2}|_{x=\xi}\frac{1}{[(x-\xi)^2-m^2\zeta]^{1/2}}\mathrm{d}\xi=-\sigma_m+\frac{1}{4\zeta}\left(\frac{\mathrm{d}\zeta}{\mathrm{d}x}\right)^2 -2\varepsilon^2\ln\frac{x+l+\sqrt{(x+l)^2-m^2\zeta}}{x+\sqrt{x^2-m^2\zeta}}\quad \text{for } Ma_\infty>1 \tag{12}$$

where $\zeta=R^2$, $\sigma_m=\frac{2}{(n-1)Ma_\infty^2}\left[1-\left(1-\frac{nMa_\infty^2}{2}\sigma\right)^{\frac{n-1}{n}}\right]$.

It is the emphasis of this paper to acquire solutions of Eqs. (11) and (12) for different Mach number, cavitation number and cone angles.

4. Discrete and iterative methods

First assuming an initial supercavity shape, the supercavity axial length is uniformly discretized into N panels, with a total number of nodes $N+1$ and $x_1=0$, $x_{N+1}=L-l$. In each panel, a locally quadratic polynomial is used to approximate ζ, that is

$$\zeta=\zeta_i+a_i(x-x_i)+b_i(x-x_i)^2,\ i=1,2,\ldots,N \tag{13}$$

where a_i and b_i are the coefficients to be determined.

According to the continuous condition (4), we have $a_1=2\varepsilon R_n$, and the recursion formula for coefficient a_{i+1} can be obtained by assuming that $d\zeta/dx$ is continuous at each discrete node, so we get

$$a_{i+1}=a_i+2b_i(x_{i+1}-x_i),\ i=1,2,\ldots,N \tag{14}$$

According to Eq. (13), the accumulation expression of ζ_k at each discrete node x_k can be written as

$$\zeta_k=\zeta_1+\sum_{i=1}^{k-1}[a_i(x_{i+1}-x_i)+b_i(x_{i+1}-x_i)^2],\ k=2,3,\ldots,N+1 \tag{15}$$

The key of computing the supercavity shape is to determine the coefficient $b_i(i=1,2,\ldots,N)$. For subsonic or supersonic flow, the substitution of Eq. (13) into Eq. (11) or Eq. (12) leads to the system