Ocean Engineering 109 (2015) 611–622

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

The bursting of a toroidal bubble at a free surface

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ARTICLE INFO

Article history:
Received 3 June 2015
Accepted 10 September 2015

Keywords:
Bubble bursting at a free surface
Bubble dynamics
Toroidal bubbles
Potential flow theory
Boundary integral methods

ABSTRACT

This paper is concerned with the bursting of a large bubble at a free surface. The numerical modelling is based on the boundary integral method. To validate the numerical model, experiments are carried out for the bursting of a spark generated bubble at a free surface in a low pressure tank, captured by using a high speed camera. Our numerical results agree qualitatively with the experiments. We further carry out numerical analysis for the bursting of an underwater explosion bubble at a free surface. We have considered the bursting of singly connected bubbles as well as toroidal bubbles. When a bubble is initiated very close to a free surface, the bursting occurs during the expansion phase, thus, resulting in a cone shaped spike at the free surface. However, when the bubble is initiated away from the free surface, it expands and collapses below the free surface and rises to the free surface due to buoyancy. An upward liquid jet forms during the later stage of collapse, which subsequently penetrates through the bubble. The toroidal bubble rises and bursts at the free surface, which results in a much higher water column due to the high speed bubble jet.

1. Introduction

The interaction of a bubble with a nearby free surface is an interesting phenomenon associated with applications in chemical engineering and bioengineering (Boulton-Stone and Blake, 1993; Handa et al., 1987; Oh et al., 1992) and underwater explosions (Barras et al., 2012; Li et al., 2012; Wang and Khoo, 2004; Whalin, 1965). This phenomenon was studied for decades using experimental (Blake and Gibson, 1987; Chahine, 1977, 1982; Zhang et al., 2013a) and numerical methods (Blake and Cerone, 1982; Blake and Gibson, 1981; Hsu et al., 2014; Klaseboer et al., 2005b).

The bursting of small bubbles at a free surface was studied by Boulton-Stone and Blake (1993) and Duchemin et al. (2002). Small bubbles can stay at the free surface in a quasi-static equilibrium for a few seconds (Blanchard and Syzdek, 1988). These simulations start as a spherical cavity connected to the outside atmosphere by a pre-existing opening in a thin liquid layer (Boulton-Stone and Blake, 1993; Duchemin et al., 2002). They noticed that a ring of fluid at the base of what was the bubble contracts to a point, throwing a plume of fluid upwards in the form of a high-speed jet.

We study the bursting of a large oscillating bubble at a free surface, subject to significant buoyancy, including their interaction before bursting and the subsequent evolution of the free surface. The phenomenon is largely affected by two dimensionless parameters: the dimensionless depth of the bubble, $\gamma = d/R_m$, and the buoyancy parameter $\delta = \sqrt{\rho g R_m/\Delta p}$, where d is the depth of the bubble centre at inception of the bubble, R_m is the maximum equivalent bubble radius and $\Delta p = p_\infty - p_v$, ρ is the fluid density, g is the gravity acceleration. Here p_∞ is the hydrostatic pressure in the undisturbed liquid and p_v is the vapour pressure. When $\gamma\delta < 0.442$, the Bjerknes force due to the free surface directed away from the free surface is predominant, the bubble moves away from the free surface and the bubble jetting is directed away from the surface too (Blake et al., 1987; Wang et al., 1996a, 1996b). When $\gamma\delta > 0.442$, the buoyancy force is predominant, the bubble rises and the bubble jetting is towards the free surface (Blake et al., 1987; Wang et al., 1996a, 1996b). A toroidal bubble forms after the jet penetrates the bubble.

When an oscillating bubble is initiated very close to a free surface, it bursts at the free surface during the expansion phase. When the bubble is not very close to the free surface, it expands and collapses beneath the free surface. If $\gamma\delta > 0.442$, the toroidal bubble rises to meet the free surface due to buoyance and then bursts at the free surface.

We will simulate both of the above two situations using the potential flow theory coupled with the boundary integral model (BIM), which is grid-free in the flow domain and has been widely used in the bubble dynamics. The BIM has been used for

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http://dx.doi.org/10.1016/j.oceaneng.2015.09.017

axisymmetric cases (Blake et al., 1997; Brujan et al., 2002; Curtiss et al., 2013; Goh et al., 2014; Guerri et al., 1981; Lind and Phillips, 2012; Liu et al., 2014; Szeri et al., 2003; Voinov, 1979; Zhang et al., 2013b, 2015b) and for three-dimensional configurations (Chahine and Perdue, 1988; Hsiao et al., 2013; Jayaprakash et al., 2012; Klaseboer et al., 2005a; Wang, 1998, 2004; Zhang and Liu, 2015).

A non-spherical bubble collapse often leads to the formation of a high-speed liquid-jet. The jet subsequently penetrates through the bubble and the liquid domain becomes doubly-connected, which results in a non-unique solution problem for a potential flow model. The doubly connected domain can be made singly-connected by, for example, using a branch cut by Best (1993) or a vortex sheet by Zhang et al. (1993) and Zhang and Duncan (1994). Wang et al. (1996b, 2005) developed a vortex ring model to model the topological transition of a bubble and the subsequent toroidal bubble dynamics. The vortex ring model is implemented in this work, since it avoids the modelling of the branch-cut or vortex sheet and the associated numerical instabilities at their intersections with the bubble surface.

Spark-generated bubbles are often used to observe bubble dynamics (Chahine and Bovis, 1980; Chahine et al., 1995; Jayaprakash et al., 2011). To validate our numerical model, we carry out experiments for a bubble bursting at a free surface. The experiment is performed in a low pressure tank (Zhang et al., 2015a), in which the air pressure above the free surface is reduced to $p_{air}=2600$ Pa$=0.026p_{atm}$, where p_{atm} is the atmospheric pressure. The bubble is generated by a low voltage electric spark generator (Zhang et al., 2013a). The low pressure tank is used to enlarge the bubble size to improve images and, more importantly, to enhance the buoyancy effects. Experiments are conducted for the bursting of both a singly-connected bubble and a toroidal bubble at a free surface. Our numerical results agree qualitatively with the experimental data for both situations.

The bursting of large underwater explosion bubbles and the water waves generated may cause significant damages to marine vessels, offshore structures and harbours (Méhauté and Wang, 1995). With the validated computational model, we simulate the bursting of an underwater explosion bubble generated by a charge with a large TNT equivalent of 1000 tons at various depths. It generates a large bubble with its maximum radius being 75 m at a depth 100 m. Analyses are carried out for the water column above the charge as well as the radial propagation of the water wave generated.

2. Mathematical and numerical model

Consider bubble dynamics near a free surface in an axisymmetric configuration. The water flow induced is assumed to be inviscid, since the Reynolds number associated is usually large. It is assumed to be incompressible, because the speed of sound in water is about 1500 ms^{-1}, the representing flow velocity is at the order $O(10)$ ms^{-1} and the Mach number is thus small (Wang and Blake, 2010, 2011; Wang, 2013, 2014). The velocity potential φ satisfies the boundary integral equation

$$c(\boldsymbol{r},t)\varphi(\boldsymbol{r},t)=\int_S\left(\frac{\partial\varphi(\boldsymbol{q},t)}{\partial n}G(\boldsymbol{r},\boldsymbol{q})-\varphi(\boldsymbol{q},t)\frac{\partial G(\boldsymbol{r},\boldsymbol{q})}{\partial n}\right)dS(\boldsymbol{q}), \tag{1}$$

where $\boldsymbol{r}$ is the field point and $\boldsymbol{q}$ is the source point, $c(\boldsymbol{r},t)$ is the solid angle and $\boldsymbol{n}$ is the unit outward normal of the boundary surface S of the flow field. The Green function is $G(\boldsymbol{r},\boldsymbol{q})=1/|\boldsymbol{r}-\boldsymbol{q}|$.

Assuming that the expansion and contraction of the bubble gas are adiabatic, the bubble pressure p is given as follows:

$$p=p_v+p_{g0}\left(\frac{V_0}{V}\right)^{\kappa}, \tag{2}$$

where V_0 and V are the initial and transient volumes of the bubble respectively, p_{g0} is the initial partial pressure of the bubble gases, and p_v is the vapour pressure inside the bubble. κ is the ratio of the specific heats of the gas, which is chosen as $\kappa=1.25$ in this work.

The boundary conditions on the bubble surface S_b and free surface S_f are as follows:

$$\frac{D\boldsymbol{r}}{Dt}=\nabla\varphi \ \text{ on } S_b \text{ and } S_f, \tag{3}$$

$$\frac{D\varphi}{Dt}=\frac{1}{2}|\nabla\varphi|^2-gz \ \text{ on } S_f, \tag{4a}$$

$$\frac{D\varphi}{Dt}=\frac{1}{2}|\nabla\varphi|^2-\frac{p-p_\infty}{\rho}-gz \ \text{ on } S_b. \tag{4b}$$

The z-axis is along the axis of symmetry. The bubble centroid at its inception is at the plane $z=0$ and p_∞ is the hydrostatic pressure on this plane.

A high-speed liquid-jet often forms and subsequently penetrates through the bubble for non-spherical collapse. The liquid domain is then transformed from a singly-connected to a doubly-connected form, which results in non-uniqueness to the potential model. The doubly-connected domain can be made singly-connected. The topological transition and the subsequent toroidal bubble dynamics will be modelled using the vortex ring model (Wang et al., 1996a, 2005).

In the vortex ring model, a vortex ring is placed inside the toroidal bubble once the jet impacts on the opposite bubble surface. The circulation Γ of the vortex-ring is equal to the jump of the potential φ at the contact point at the moment of impact. The potential φ is then decomposed as follows:

$$\varphi=\varphi_{vr}+\phi, \tag{5}$$

where φ_{vr} is the potential due to the vortex ring and ϕ is the remaining potential.

The boundary conditions (4a,b) can be written in terms of the remaining potential ϕ as follows:

$$\frac{D\phi}{Dt}=\frac{1}{2}|\boldsymbol{v}_{vr}+\nabla\phi|^2-\boldsymbol{v}_{vr}\cdot(\boldsymbol{v}_{vr}+\nabla\phi)-gz \ \text{ on } S_f, \tag{6a}$$

$$\frac{D\phi}{Dt}=\frac{1}{2}\left|\boldsymbol{v}_{vr}+\nabla\phi\right|^2-\boldsymbol{v}_{vr}\cdot(\boldsymbol{v}_{vr}+\nabla\phi)-\frac{p-p_\infty}{\rho}-gz \text{ on } S_b, \tag{6b}$$

where $\boldsymbol{v}_{vr}=\nabla\varphi_{vr}$. The details on the vortex ring modelling for toroidal bubbles are referenced to Wang et al. (1996a).

The integration in time is carried out using a variable time step (Blake and Gibson, 1987):

$$\Delta t=R_m\left(\frac{\Delta p}{\rho}\right)^{0.5}\frac{\Delta\Phi}{\max\left|0.5|\nabla\varphi|^2+gz+\frac{p+p_\infty}{\rho}\right|}, \tag{7}$$

where $\Delta\Phi > 0$ is a constant, which was chosen as 0.02 in the calculations presented in this paper.

At each time step, we have a known bubble surface, a known free surface and known potential distributions on them. With this information we can calculate the tangential velocity on the two surfaces. The normal velocities on them are obtained by solving the boundary integral equation (1). The bubble and free surfaces and the potential distributions on them can be further updated by performing the Lagrangian time integration to (3) and (4a,4b), respectively. The details on the numerical model using the BIM for the problem can be found in Wang and Manmi (2014, 2015).