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Numerical study of nonlinear Peregrine breather under finite water depth

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ABSTRACT

Though the focusing method can effectively generate waves which satisfy the definition of rogue waves at a specific position and moment, however, being inherently not nonlinear, the focusing model is still a controversial rogue wave generation method. Recently, nonlinear models are becoming more popular for rogue wave generation in physical and numerical tanks. In this paper, a weakly nonlinear model known as the Peregrine breather solution of the cubic Schrödinger equation is studied under finite water depth. In contrast with the focusing model, nonlinearity is considered throughout the simulation process, i.e., from the wave model to the generated waves. The numerical results are validated against theoretical solutions as well as experimental measurements. To further investigate their temporal-frequency characteristics, a wavelet analysis is performed on the generated Peregrine breather, and the concepts of life time and traveling distance are studied. The influence of higher order nonlinearity, i.e., the 2nd-order Stokes component of the perturbed expansion under finite water depth, is taken into account. We also discuss the influence of 2nd-order term on the life time, travel distance, and energy distribution.

1. Introduction

Rogue waves, also named freak waves, are giant ocean waves which induce great hazards on the safety of ships and marine structures. Usually, rogue waves are defined as waves whose heights exceed the significant wave height in 2 times (Kjeldsen, 2005; Sand et al., 1990). For years, researchers have been working on the physical mechanisms of rogue waves. Although linear models give a simple and intuitive grasp of rogue waves, nonlinear models play an important role in explaining their physical mechanisms due to the giant crest and steepness. Kharif and Pelinovsky (2003) and Pelinovsky and Kharif (2008) gave a comprehensive overview of the existing rogue-wave models. Categorized by their governing equations, the nonlinear models can be divided into several types, i.e., the models based on the nonlinear Schrödinger (NLS) equation under finite or deep water depth (Zakharov et al., 2006), the Korteweg-de Vries (KdV) equation under shallow water depth (Kit et al., 2000), and more universally, the Laplace equation or the Navier–Stokes (N–S) equations, etc. The NLS equation is a well-known equation not only in wave mechanics, but also in optics and quantum mechanics, and has been thoroughly studied both analytically and numerically. On the basis of the NLS equation, rogue waves can be explained as driven by modulational (Benjamin–Feir) instability (Zakharov et al., 2006; Osborne et al., 2000) or some breather solutions which contain nonlinear focusing effects (Peregrine, 1983).

Apart from their physical mechanisms, researchers are also studying rogue wave generations in laboratories and numerical tanks. The superposition-based model is a widely utilized method under laboratories and numerical wave flumes (Cui et al., 2013; Zhao et al., 2010; Cui et al., 2011; Fochesato et al., 2007; Sun et al., 2009; Liu et al., 2011). In the superposition model, the rogue wave is treated as the superposition of wave components with various frequencies and phases. With these wave components, the paddle movement signal is obtained based on a linear wave-maker transfer function (Dean and Dalrymple, 1991). By adjusting the phases and energy proportions of these wave components, giving a huge crest at a designated location and moment, a giant wave or rogue wave could be generated. Sometimes, a focusing wave train is appended to the random wave field to improve the simulation efficiency. Despite the superposition-based model is capable of generating large waves which satisfy the definition of rogue waves (Cui et al., 2013), the assumption underlying the superposition of waves is the linear free surface condition. In addition, the wave-maker transfer function is also essentially linear. Thereby, the superposition-based model sometimes gives rise to controversies

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as it fails to take into account the nonlinearity of rogue-wave phenomena.

To avoid the dispute mentioned above, more researchers have turned to rogue-wave generation in a nonlinear manner, based on nonlinear models such as NLS, KdV and N–S equations. A prevalent model is the Peregrine breather solution of the cubic NLS equation, which was first deduced by Peregrine (1983) on the foundation of the solution given by Ma (1979). Chabchoub et al. (2011, 2012a, 2012b) utilized the Peregrine breather to generate deep-water rogue waves in laboratory. Onorato et al. (2013), using the Peregrine breather, simulated a rogue wave which is similar to the famous "New Year Wave" (Haver and Andersen, 2000), and performed a sea-keeping test on a chemical tanker. Comparisons against theoretical solutions obtained by Chabchoub et al. (2011, 2012a, 2012b) and Onorato et al. (2013) showed that the Peregrine breather can be properly simulated in laboratory. However, small mismatches could still be observed, especially between the gradients in amplitude modulations (Chabchoub et al., 2012a). Perić et al. (2015) analyzed the Peregrine breather by direct numerical simulations on the two-phase Navier–Stokes model, with a VOF method applied to rogue wave dynamics up to the initial stages of wave breaking. Didenkulova et al. (2013) demonstrated that the rogue wave packet becomes wider and contains more individual waves in intermediate rather than in deep waters. One difficulty has to be overcome for rogue-wave generation using nonlinear models, i.e., the motion of wave paddle has to be carefully designed to create a nonlinear rogue wave within a specific region.

The experiments of Chabchoub et al. (2011, 2012a) and Onorato et al. (2013) are based on the 1st-order Peregrine breather wave. Recently, researchers have been working to generate high-order rogue waves, for instance the high-order Peregrine solutions (also known as Akhmediev–Peregrine breathers), using tools such as the Darboux transformation (Xu et al., 2011). Slunyaev et al. (2013) studied rational solutions using models of Dysthe equation and potential Euler equations. Chabchoub et al. (2012b) observed a hierarchy of up to fifth-order deep-water rogue waves in a water tank. He et al. (2013) and Zhang et al. (2014) introduced a mechanism for generating higher-order rogue waves (HRWs) of the NLS equation. Though high-order deep-water rogue waves have been adequately studied both theoretically and experimentally, the study of high-order rogue waves under finite water depth is still not much.

In this paper, a series of numerical simulations are performed on the Peregrine breather solution of the cubic NLS equation, using a 2-D numerical wave channel based on the incompressible N–S equations. First, the primary harmonic, i.e., the 1st-order Stokes component is simulated. In order to validate our numerical settings, the numerical results are compared against the experimental measurements of Chabchoub et al. (2011, 2012a). To describe the generated Peregrine breathers, we employ the life time and travelling distance proposed by Chabchoub et al. (2012a). Then, the expression of 2nd-order harmonic, as well as the corresponding particle velocity and pressure are deduced, which is not considered in Chabchoub's and Onorato's experiments. The 2nd-order Stokes component is generated in a numerical channel, and is validated against the theoretical expression. The 2nd-order harmonic envelope is studied to analyze the high-order nonlinear effect on the wave contour. Comparisons are also performed to investigate the influence of higher nonlinearity on the Peregrine breather surface history, as well as its life time and travelling distance. During simulations, the energy spectrum is obtained by performing a Discrete Fourier Transformation (DFT) on the surface elevation histories, and a wavelet analysis is conducted to further reveal the temporal-frequency distribution of wave energy.

2. Peregrine breather wave

In order to investigate the scales and nonlinearities of various degrees, usually a multiple scale perturbation expansion, formulated as (1), is performed on the original Euler equation as follows:

$$\phi = \sum_{n=1} \varepsilon^n \phi_n, \quad \zeta = \sum_{n=1} \varepsilon^n \zeta_n \tag{1}$$

Here ϕ_n and ζ_n are the velocity potential and surface elevation of the n-th order (or the n-th order Stokes component). $\varepsilon = kA_0$ is a small parameter, where k and A_0 are the wave number and amplitude of carrier waves, respectively. ϕ_n and ζ_n are functions of coordinates under various scales, which are written as

$$\begin{cases} \phi_n = \phi_n(x, x_1, x_2, \ldots; z; t, t_1, t_2, \ldots) \\ \zeta_n = \zeta_n(x, x_1, x_2, \ldots; t, t_1, t_2, \ldots) \end{cases} \tag{2}$$

where the scaled coordinates are

$$\begin{cases} x, x_1 = \varepsilon x, \quad x_2 = \varepsilon^2 x, \quad \ldots \\ t, t_1 = \varepsilon t, \quad t_2 = \varepsilon^2 t, \quad \ldots \end{cases} \tag{3}$$

Though the potential component ϕ_n is of order n, it contains harmonics of various orders, and is further formulated as

$$\phi_n = \sum_{m=-n}^{n} e^{im\psi} \phi_{nm} \tag{4}$$

where i is the imaginary unit, $\psi = kx - \omega t$ is the wave phase, and ω is the angular frequency of carrier waves.

Based on Eqs. (1)–(4), the Euler equation can be expanded into a series of equations, which corresponds to different orders and harmonics. By solving these equations, the velocity potential and surface elevation of each order and harmonic could be obtained. These expansions and equations are not hard to deduce, but the deduction is tedious and interminable. The detailed derivation could be found in Mei (1989), and we only refer to some useful expressions in this paper.

It should be mentioned, the following terms are adopted hereafter to make our discussions brief and clear: 1st-order Stokes component, i.e., ζ_1 of Eq. (1); 2nd-order Stokes component, i.e., ζ_2 of Eq. (1); 1st-order surface elevation, i.e., $\zeta = \varepsilon\zeta_1$; 2nd-order surface elevation, i.e., $\zeta = \varepsilon\zeta_1 + \varepsilon^2\zeta_2$.

2.1. 1st-order Stokes component

The 1st-order Stokes component only considers the first term of Eq. (1), and is written as

$$\phi_1 = \phi_{10} - \frac{g \cosh Q}{2\omega \cosh q}\left(iAe^{i\psi} + c.c.\right) \tag{5}$$

$$\zeta_1 = \frac{1}{2}\left(Ae^{i\psi} + c.c.\right) \tag{6}$$

where $Q = k(z+h)$, $q = kh$, h is the water depth, g is the gravitational acceleration, $c.c.$ denotes the complex conjugate. It should be noted that the origin of coordinate is positioned on the still water surface, thus $z = 0$ represents mean water surface and $z = -h$ the bottom. The dispersion relationship is $\omega^2 = gk \tanh(kh)$. ϕ_{10} is a "constant" term for the 1st-order potential, which describes the mean flow, and A is the envelope of carrier waves. Though being constants under a normal scale, ϕ_{10} and A could slowly change over a larger scale $t_1, t_2, \ldots$ and $x_1, x_2, \ldots$, governed by the equations below

$$\frac{\partial \phi_{10}}{\partial \xi} = -\frac{\omega^2\left(2\omega\cosh^2 q + kC_g\right)}{4k\sinh^2 q\left(gh - C_g\right)}|A|^2 \tag{7}$$