Ocean Engineering 108 (2015) 241–256

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Free and forced vibration of ring-stiffened conical–cylindrical shells with arbitrary boundary conditions

Meixia Chen, Kun Xie*, Wenchao Jia, Kun Xu

School of Naval Architecture and Ocean Engineering, Huazhong University of Science and Technology, 1037 Luoyu Road, Wuhan 430074, China

ARTICLE INFO

Article history:
Received 25 March 2015
Accepted 31 July 2015

Keywords:
Combined conical–cylindrical shell
Stiffeners
Bulkheads
Vibration analysis
Arbitrary boundary conditions

ABSTRACT

An analytic method is presented to analyze free and forced vibration characteristics of ring-stiffened combined conical–cylindrical shells with arbitrary boundary conditions, e.g. classical and elastic ones. The combined shell is firstly divided into multiple substructures according to the junctions of shell–shell and shell–plate, and/or the location of driving point. Then, Flügge theory is adopted to describe the motions of the cylindrical and conical segments. Instead of adopting the smeared out method and treating the ring stiffeners as beams, the stiffeners with rectangular cross-section are treated as discrete members and the equations of motion of annular plate are used to describe the motion of stiffeners. Power series, wave functions and Bessel functions are used to express the displacement functions of conical segment, cylindrical segment and annular plate, respectively. Lastly, boundary conditions and continuity conditions between adjacent substructures are used to assemble the final governing equation. Results of present method show good agreement with the results in literature and the results calculated by finite element method (FEM). In addition, the influences of boundary conditions and ring stiffeners on the free vibration are studied. The effects of direction of external force and bulkheads on the forced vibration are also discussed.

1. Introduction

The combined conical–cylindrical shells are widely used in engineering application, such as water tanks, aircraft, submarine and so on. The vibration characteristics of individual shells, e.g. cylindrical and conical shells, have been deeply studied by lots of researchers and those works were well summarized (Leissa, 1993; Qatu 2002a, 2002b). However, due to the complexity involved in the modeling and solution process, the research on the combined conical–cylindrical shells is very little. On the other hand, the vibration characteristics of the combined shells are of great influence on their performance and special attentions are required in analysis and design process. In this context, developing an efficient approach to evaluate the vibration characteristics of the combined shells is meaningful.

The vibration characteristics of combined shells have been investigated since the ’60s. The early methods analyzing vibration of combined shells include multi-segmental numerical integration technique (Hu and Raney, 1967), variational finite-differences method (Galletly and Mistry, 1974), transfer matrix method (Irie et al., 1984), state space method (Tavakoli and Singh, 1989), semi-analytical finite element method (Sivadas and Ganesan, 1993) and so forth. In recent years, since the widely engineering application of the combined shells and the highly reliability requirement, more and more scholars have shown extensive interest in the research of the vibration characteristics of combined shells. Finite element method is one of the most extensive methods to analyze the vibration of combined shells (Benjeddou, 2000; Patel et al., 2000; EI Damatty et al., 2005). Dynamic stiffness matrix was employed by Efraim and Eisenberger (2006) to solve the exact vibration frequencies of segmented shells of revolution. Caresta and Kessissoglou (2010a) combined power series method with wave propagation method to study the free vibration of isotropic coupled cylindrical–conical shells. Qu et al. (2013a, 2013b) proposed a variational method to study the vibration of joined cylindrical–conical shells with classical and non-classical boundary conditions. In those two papers, the hull needed to be divided into multiple segments, and the number of segments directly affected the convergence of results. Besides, the value of weighted parameter, which was introduced to assemble adjacent segments, was of great influence on the accuracy of results. A modified Fourier–Ritz method was developed by Jin et al. (2014a,2014b,2014c) and Su et al. (2014) to analyze free vibration of plates and shells with generate boundary conditions. Then Ma

* Corresponding author. Tel: +86 15927457822.
E-mail address: xiekun79@163.com (K. Xie).

http://dx.doi.org/10.1016/j.oceaneng.2015.07.065
0029-8018/

et al. (2014) extended the method to analyze free and forced vibration characteristics of coupled conical–cylindrical shells with arbitrary boundary conditions. However, the results of elastic boundary were not validated and no ring stiffeners were considered. Lee et al. (2002) adopted Rayleigh–Ritz method to investigate the free vibration characteristics of joined spherical–cylindrical shell, but the hemispherical shell was assumed to have free boundary condition at the join part while the boundary of the cylindrical shell was simply supported. Lee and Choi (2001) used receptance method to analyze free vibrations of simply supported cylindrical shells with an interior rectangular plate. Based on transfer matrix method, Liang and Chen (2006) investigated the natural frequencies and mode shapes for a conical shell with an annular end plate.

From above analysis it's known that the emphasis in the above cited references is the free vibration of combined shells without stiffeners, and literature about the free and forced vibration of ring-stiffened conical–cylindrical shell is much less. However, the ring stiffeners are extensively used to strengthen shells and they are of great effect on the vibration characteristics of the shells. Customarily, the effect of stiffeners is studied either by applying "smeared out" technique (Hoppmann, 1958; Caresta et al, 2008; Caresta and Kessissoglou, 2010b) or by treating stiffeners as discrete member (Thein and Hu, 1967; Basdekas and Chi, 1971; Jafari and Bagheri, 2006; Chen et al., 2013; Wei et al. 2013). The former approach is suitable for equal spacing, small size and dense stiffeners while treating stiffeners as discrete members can be used to analyze stiffeners with non-uniform distributions and sizes. Beam model was used to describe the motion of the ring stiffeners (Thein and Hu, 1967; Basdekas and Chi, 1971; Jafari and Bagheri, 2006) and it could lead to inaccurate results when the size of stiffener is large or the eccentricity of stiffeners is non-zero. In addition, Donnell–Mushtari theory was employed by Thein and Hu (1967) and it leaded to inaccurate results for stiffened cylindrical shells at low frequencies (Ruotolo, 2001). On the other hand, the research group of the authors treated ring stiffeners with rectangular cross-section as annular plates, and the comparison of the results of different methods showed that the method was a very accurate method dealing with ring stiffeners (Chen et al., 2013; Wei et al., 2013). Besides ring stiffeners, the bulkheads are also widely used in the shells to satisfy practical engineering applications, and there is almost no literature about the effect of bulkheads on vibration of the ring-stiffened combined conical–cylindrical shell.

In the present paper, an analytic method is presented to analyze free and forced vibration of combined conical–cylindrical shells with arbitrary boundary conditions, including classical and elastic boundary conditions. Firstly, the ring-stiffened conical–cylindrical shell is divided into multiple substructures according to the junctions of shell–shell and shell–plate, and/or the locations of driving points. Then, Flügge theory is adopted to describe the motions of the shell segments and the equations of motion of annular plate are used to describe the motions of stiffeners. The displacement functions of conical segments, cylindrical segments and annular plates are expanded in power series, wave functions and Bessel functions, respectively. The boundary conditions and

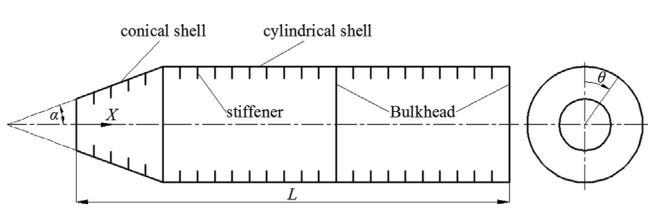

Fig. 1. Coordinate system of a conical–cylindrical shell with ring stiffeners and bulkheads.

interface continuity conditions between adjacent substructures are utilized to assemble the final governing equation. In contrast with FEM, the present method has advantages in rapid convergence and small size of governing equation, which results in higher computational efficiency. On the other hand, the present method provides more physical insight into the vibration behaviors since the displacement functions in all substructures are expanded in terms of circumferential mode number.

2. Theory formulations

Fig. 1 shows the global coordinate system for a combined conical–cylindrical shell with stiffeners and bulkheads, (X, θ). θ is the circumferential coordinate and it is consistent in all substructures. The axial coordinate is indicated by capital letter X, which is different from the local axial or meridional coordinates of substructures. L is the total axial length of combined shell. To obtain the final governing equation of the combined shell, the hull is firstly divided into many substructures according to the junctions of shell–shell and shell–plate, and/or location of the driving point. When studying the forced vibration of the combined shell, to simplify the presentation, only one external point force located at one junction of shell–shell or shell–plate is considered in the following theoretical deduction. Nevertheless, it can be easily extended to discuss more than one external point excitation located at any part of the combined shell.

2.1. Conical shells

Fig. 2 shows the local coordinate system, displacement and force resultants of a conical shell. The local coordinate system is (x_c, θ_c), and the meridional coordinate x_c is measured from the middle of conical shell. R_1 and R_2 are the radii of small and large ends, respectively. R_0 is the mean radius and R_c is the radius at location x_c. α is the semi-vertex angle. u_c, v_c and w_c are the orthogonal components of displacement in x_c, θ_c and z_c directions. β_c denotes the slope. M_c is the bending moment resultant, N_c is the meridional force resultant (along generatrix direction), $\overline{S}_c$ and $\overline{T}_c$ are the normal and circumferential Kelvin–Kirchhoff shear force resultants, respectively. The detailed expressions of $M_c, \overline{S}_c, \overline{T}_c, N_c$ are given in Appendix A.

The motion of thin conical shells can be described by Flügge theory and the corresponding equations are (Leissa, 1993)

$$\begin{cases} L_{c,11}u_c + L_{c,12}v_c + L_{c,13}w_c = 0 \\ L_{c,21}u_c + L_{c,22}v_c + L_{c,23}w_c = 0 \\ L_{c,31}u_c + L_{32}v_c + L_{c,33}w_c = 0 \end{cases} \tag{1}$$

where differential operators $L_{c,ij} (i, j = 1, 2, 3)$ of conical shell are also given in Appendix A.

Eq. (1) can be solved by using power series approach proposed by Tong (1993). After mathematical deduction, corresponding solutions can be expressed as (Caresta, 2009)

$$\begin{cases} u_c(x,\theta,t) = \sum_{n=0}^{\infty} \mathbf{u}_{c,n} \cdot \mathbf{x}_{c,n} \cos(n\theta) e^{-j\omega t} \\ v_c(x,\theta,t) = \sum_{n=0}^{\infty} \mathbf{v}_{c,n} \cdot \mathbf{x}_{c,n} \sin(n\theta) e^{-j\omega t} \\ w_c(x,\theta,t) = \sum_{n=0}^{\infty} \mathbf{w}_{c,n} \cdot \mathbf{x}_{c,n} \cos(n\theta) e^{-j\omega t} \end{cases} \tag{2}$$

where n is the circumferential mode number, ω is the circular frequency and t is the time. The vectors $\mathbf{u}_{c,n}, \mathbf{v}_{c,n}, \mathbf{w}_{c,n}$ are

$$\begin{cases} \mathbf{u}_{c,n} = [u_{c1}(x) \ldots u_{c8}(x)] \\ \mathbf{v}_{c,n} = [v_{c1}(x) \ldots v_{c8}(x)] \\ \mathbf{w}_{c,n} = [w_{c1}(x) \ldots w_{c8}(x)] \end{cases} \tag{3}$$