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Research of complex modal parameters extraction of a multi-degree-of-freedom structure based on similarity search

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ABSTRACT

We present a method for the extraction of modal parameters that could be useful for the parameter identification of complex structural dynamic modes of multi degree of freedom systems that may be subject to environmental load excitations. The approach is linear and in this sense it operates on the basis of the principles suggested by the so-called Rayleigh superposition method. This atoms which have the form of free vibration are constructed. The method combined with MP (Matching Putsuit) and GA (Genetic Algorithm). MP is borrowed to extract every most similar atom to the random decrement of the structure response to identify modal parameters. GA is used to speed up this process. The method is applied on the analysis of model experiments for an FPSO single point mooring system and results are compared against real monitoring data. It is shown that the method is capable of extracting successfully modal characteristics such as complex modal frequency, damping ratio and mode shapes.

1. Introduction

Generally, the identification of the structure modal parameters based on environmental loads can only rely on the monitoring data of the structural response. The free response is constructed by the RDT (random decrement technique) and is used to match the free response formula of the theory to identify the structure modal parameters. The common method to identify the modal parameters is based on a time-domain, such as, the LSCE (Least Square Complex Exponential) method (Brown et al., 1979), ARMA models (Akaike, 1969), ITD (Ibrahim Time Domain) method (Ibrahim, 1973, 1987), and SSI (Stochastic Subspace Identification) (Bakir, 2011). The method based on a time domain is sensitive to noise and has a high requirement for stationarity. The time–frequency identification is also a hot spot and has made a lot progress, of which the modal parameter estimation based on wavelet transformation (Le and Paultre, 2012) and HHT (Hilbert–Huang Transform) identification (Huang et al., 1998) are representative methods. Both these two methods decompose the data into several models. The data are decomposed into a multi-resolution view based on the wavelet basis, and the modal parameters are identified by the comparison of the theoretical formula. HHT decomposes the data into simple IMF components using EMD while the instantaneous frequency can then be identified by the free response that has been Hilbert transformed. The advantages of both methods are the strong noise immunity and filtering capacity of the non-stationary component in the signal. It is significant in engineering since this method resolves the problem of non-linear modal identification which the time-domain methods cannot do. By using experimental test data and the methods of model updating, models of the structure can be improved to predict structure behavior correctly. There have been many mature algorithms of model updating (Friswell and Mottershead, 1995). Two critical issues: how a finite-element model should be parameterized and estimating the unknown parameters from the resulting ill-conditioned equations in model updating are discussed (Friswell et al., 2001). In order to eliminate model error which may give rise to inaccuracy in the model predictions, test data is used to update initial model. It is an effective method to obtain high accuracy finite-element model (Mottershead and Friswell, 1993). Both the time–frequency method and the time-domain method are analytic methods, that is to say, calculating matrices and transforming signal to obtain the modal parameters of a structure.

Modal parameter extraction based on similarity search is a non-analytic method. The basic idea of this method is to search for the most similar atoms with a free response structure in the dictionaries which have been constructed and then extract the modal parameters. MP is a common method in similarity search (Mallat et al., 1993). Jiang et al. decompose the original signal into simple IMF components by

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using EMD, and then match the IMFs with the given atoms separately to extract the modal parameters (Jiang et al., 2010). Zhu et al. propose using the Laplace wavelet function for the dictionary atoms, introduce PSO (Particle Swarm Optimization) to accelerate the search for the most similar atom. As a result, the frequency and damping ratio of the device can be effectively extracted (Zhu et al., 2007). Dong et al. and Xun et al. use damped sinusoidal functions and Gabor functions as the dictionary atoms respectively to search for the most similar atoms compared with the original signal and to identify the low-frequency oscillation and subsynchronous oscillations modal parameters in the power system (Dong et al., 2013; Xun et al., 2013). Modal extraction based on similarity search is characterized by the simplicity of the method, as well as the precision in the extracting modal frequency and damping ratio. However, the extraction of modal parameters based on the similarity search belongs to the partial method, i.e., extracting multiple modal parameters of the structure from a set of data. Consequently, the overall characteristics of the structure, for example, the modal shape, cannot be obtained. In addition, the phase information obtained by search has no practical meaning.

In consideration of the identified problem mentioned above, a method of Multi-DOF structure complex modal parameter extraction based on similarity search is proposed in this paper. This method is based on the Multi-DOF complex modal theory of the general viscous system in which the most similar atoms with a free response structure are selected from the constructed dictionary so that the goal of identifying the modal parameters of the Multi-DOF structure has been achieved. The complex modal frequency, damping ratio and modal shape can be globally extracted by using this method with the measurement data of the Multi-DOF displacement. Furthermore, the initial phase has practical meaning for this method. The MP and the GA are borrowed into the method to perform the similarity comparison and reduce the time complexity of the search process. Simulation and experiment results show that the method proposed in this paper can identify the complex modal frequency, damping ratio and modal shape with high precision.

2. The theory of the modal shape superposition of the complex mode structure

The vibration differential equation of the general viscous damping (Peiying et al., 2008) is

$$\mathbf{M}\ddot{\mathbf{x}}+\mathbf{C}\dot{\mathbf{x}}+\mathbf{K}\mathbf{x}=\mathbf{f}(t) \tag{1}$$

where $\mathbf{C}$ is the general viscous damping matrix, which cannot be diagonalized by the modal matrix of the undamped system. Eq. (1) can be written as follows:

$$\begin{bmatrix}\mathbf{C} & \mathbf{M}\\ \mathbf{M} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\dot{\mathbf{x}}\\ \ddot{\mathbf{x}}\end{bmatrix}+\begin{bmatrix}\mathbf{K} & \mathbf{0}\\ \mathbf{0} & -\mathbf{M}\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \dot{\mathbf{x}}\end{bmatrix}=\begin{bmatrix}\mathbf{f}(t)\\ 0\end{bmatrix} \tag{2}$$

When $\mathbf{f}(t)=0$, the generalized eigenvalue problem of Eq. (2) can be expressed as

$$\left(\lambda\begin{bmatrix}\mathbf{C} & \mathbf{M}\\ \mathbf{M} & \mathbf{0}\end{bmatrix}+\begin{bmatrix}\mathbf{K} & \mathbf{0}\\ \mathbf{0} & -\mathbf{M}\end{bmatrix}\right)\Phi'=0 \tag{3}$$

The generalized eigenvalue in the conjugate form resulting from Eq. (3) is

$$\begin{cases}\lambda_i=-\sigma_{mi}+j\omega_{mdi}=-\zeta_{mi}\omega_{mi}+j\omega_{mi}\sqrt{1-\zeta_{mi}^2}\\ \lambda_i^*=-\sigma_{mi}-j\omega_{mdi}=-\zeta_{mi}\omega_{mi}-j\omega_{mi}\sqrt{1-\zeta_{mi}^2}\end{cases}\quad(i=1,2,\ldots,N) \tag{4}$$

where σ_{mi} is the ith complex modal damping coefficient of the generalized viscous system, $\omega_{mi}=|\lambda_i|$ is the ith complex modal natural frequency, $\zeta_{mi}=\sigma_{mi}/\omega_{mi}$ is the ith complex modal damping ratio, and $\omega_{mdi}=\sqrt{\omega_{mi}^2-\sigma_{mi}^2}=\omega_{mi}\sqrt{1-\zeta_{mi}^2}$ is the ith complex modal damping natural frequency of the generalized viscous system. The corresponding conjugate complex eigenvectors are obtained as follows:

$$\Phi_i'=\begin{bmatrix}\Phi_i\\ \lambda_i\Phi_i\end{bmatrix},\quad \Phi_i'{*}=\begin{bmatrix}\Phi_i^*\\ \lambda_i^*\Phi_i^*\end{bmatrix}\quad(i=1,2,\ldots,N) \tag{5}$$

The orthogonality of the eigenvectors of the generalized viscous damping system can be expressed as

$$\left.\begin{aligned}&\Phi'^{\mathbf{T}}\mathbf{P}\Phi'=diag[a_i,a_i^*]\\ &\Phi^{prime\mathbf{T}}\mathbf{Q}\Phi'=diag[b_i,b_i^*]\end{aligned}\right\} \tag{6}$$

here $\mathbf{\Phi}'=\begin{bmatrix}\mathbf{\Phi} & \mathbf{\Phi}^*\\ \mathbf{\Lambda\Phi} & \mathbf{\Lambda}^*\mathbf{\Phi}^*\end{bmatrix}$, $\mathbf{\Lambda}=diag[\lambda_i]$, $\mathbf{\Lambda}^*=diag[\lambda_i^*]$, and $\lambda_i=-\frac{b_i}{a_i}$, $\lambda_i^*=-\frac{b_i^*}{a_i^*}$ $(i=1,2,\ldots,N)$. Given the orthogonality of the complex eigenvec-
tor Φ_i', $\Phi_i'^*$, the complete orthogonal basis of the complex vector space in the $2N$ dimension is formed by the complex vectors of the linear independence of $2N$, thus

$$\begin{bmatrix}\mathbf{x}\\ \dot{\mathbf{x}}\end{bmatrix}=\begin{bmatrix}\Phi & \Phi^*\\ \Lambda\Phi & \Lambda^*\Phi^*\end{bmatrix}\begin{bmatrix}\mathbf{y}\\ \mathbf{y}^*\end{bmatrix} \tag{7}$$

where $[\mathbf{y}\ \mathbf{y}^*]^T$ is the coordinate vector of $[\mathbf{x}\ \dot{\mathbf{x}}]^T$ in this complex vector space.

When $\mathbf{f}(t)=0$, combing Eqs. (7) and (2) and using the orthogonality of the complex eigenvector, $[\mathbf{y}\ \mathbf{y}^*]^T$ is as follows:

$$\begin{bmatrix}\mathbf{y}\\ \mathbf{y}^*\end{bmatrix}=diag[e^{\lambda_i t},e^{\lambda_i^* t}]\begin{bmatrix}\mathbf{y}(0)\\ \mathbf{y}^*(0)\end{bmatrix}$$

where $\begin{bmatrix}\mathbf{y}(0)\\ \mathbf{y}^*(0)\end{bmatrix}=\begin{bmatrix}\Phi & \Phi^*\\ \Lambda\Phi & \Lambda^*\Phi^*\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{x}(0)\\ \dot{\mathbf{x}}(0)\end{bmatrix}$ is related to the initial conditions.

Accordingly, the displacement free response of Eq. (2) in the physical coordinates is obtained as follows:

$$\mathbf{x}=\Phi diag[e^{\lambda_i t}]\mathbf{y}(0)+\Phi^* diag[e^{\lambda_i^* t}]\mathbf{y}^*(0)=\sum_{i=1}^{N}\left(\Phi_i e^{\lambda_i t}y_i(0)+\Phi_i^* e^{\lambda_i^* t}y_i^*(0)\right) \tag{8}$$

Assuming that $y_i(0)=T_ie^{j\varphi_i}$ and $y_i^*(0)=T_ie^{-j\varphi_i}$, $\mathbf{x}$ is as follows:

$$\mathbf{x}=\sum_{i=1}^{N}T_ie^{-\sigma_{mi}t}(\Phi_ie^{j(\omega_{mdi}t+\varphi_i)}+\Phi_i^*e^{-j(\omega_{mdi}t+\varphi_i)}) \tag{9}$$

when the system has the vibration in one order complex frequency ω_{mdi}, the law of vibration is

$$\mathbf{x}_i=T_ie^{-\sigma_{mi}t}(\Phi_ie^{j(\omega_{mdi}t+\varphi_i)}+\Phi_i^*e^{-j(\omega_{mdi}t+\varphi_i)}) \tag{10}$$

where

$$x_{ji}=T_ie^{-\sigma_{mi}t}(\phi_{ji}e^{j(\omega_{mdi}t+\varphi_i)}+\phi_{ji}^*e^{-j(\omega_{mdi}t+\varphi_i)})\quad(i,j=1,2,\ldots,N) \tag{11}$$

assuming

$$\phi_{ji}=\eta_{ji}e^{j\gamma_{ji}},\quad \phi_{ji}^*=\eta_{ji}e^{-j\gamma_{ji}}\quad(i,j=1,2,\ldots,N) \tag{12}$$

thus

$$x_{ji}=2T_i\eta_{ji}e^{-\sigma_{mi}t}\cos(\omega_{mdi}t+\varphi_i+\gamma_{ji})\quad(i,j=1,2,\ldots,N) \tag{13}$$

It is clear that when the generalized viscous damping system does have a free vibration in the ith order frequency ω_{mdi}, the initial phase of the jth physical coordinate is $\varphi_i+\gamma_{ji}$. The initial phase is not only related to i but also related to the physical coordinate j, which means each component of the modal shape has a different initial phase.