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Nonlinear dynamics of an underwater slender beam with two axially moving supports

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ABSTRACT

This paper investigates the nonlinear dynamic behavior of a towed underwater beam with two supported ends. The equation of motion is derived by the Newtonian approach. An "axial added mass coefficient" is taken into account to get a better approximation for the mass of fluid attached to beams. Nonlinear deflection-dependent axial forces are also considered. The dynamics of the system is studied via Galerkin approach and Runge-Kutta technique. The linear dynamic analysis is conducted firstly. The solution for natural frequency is obtained and the result shows that the beam will subject to buckling-type instability if the moving speed exceeds a certain value. Then, the buckled configuration is obtained and its stability is discussed in the nonlinear dynamic analysis. It is found that the subcritical Hopf bifurcation of the first buckled mode may occur when the towing speed reaches to a critical value. In addition, the nonlinear dynamic responses are calculated and the periodic-1, period-3, period-5, quasi-periodic and chaotic motions are detected. Meanwhile, the result shows the route to chaos for the beam is via period-3 motions or quasi-periodic motions. The effects of several system parameters on the chaotic motion are also studied.

1. Introduction

The dynamics of underwater towed systems has attracted much attention because of its wide application, such as slender structures towed to the location of installation (Sarv and John, 2000; Kyriakides and Corona, 2007). It is noted that the underwater towed slender structure belongs to axially moving systems since the direction of the towed motions is along the axial of the slender structure in general. Meanwhile, the axially moving systems can be grouped into two types: first, systems with fixed and unmovable support; and second, systems that the axial motion of the structures is induced by the axial movements of the supports (Ni et al., 2014). Obviously, the underwater towed slender structure belongs to the second type, i.e. axially moving systems.

The linear and nonlinear dynamics of the first type of axially moving materials, including band saw (Mote, 1965) and axially moving elastic beams (Öz, 2001; Ding and Chen, 2009; Ghayesh, 2012), have been investigated by many researchers. To date, only few literatures can be found on the first type of axially moving systems with the consideration of fluid-structure interactions. The dynamics of a cantilever beam being deployed in a dense incompressible fluid was explored by Taleb and Misra (1981). However, the fluid-dynamic forces were not correctly accounted for in the analysis performed by Taleb and Misra (1981), as proved by Gosselin et al. (2007). As stated by Gosselin et al. (2007), in the axial direction of the beam, the layer of fluid which stays attached to the beam is in fact considerably smaller than that of this lateral-direction virtual mass. Thus, an "axial added mass coefficient" was introduced in order to better approximate the mass of fluid which stays attached to the oscillating beam while moving in the axial direction. With axial added mass coefficient taken into account, Wang and Ni (2008) investigated the linear vibration and stability of the first type axially moving pinned-pinned, clamped-clamped and clamped-pinned beam immersed in fluid by DQM.

The studies on the dynamics of the second type of axially moving systems are mainly focused on the underwater towed systems because of its wide application in ocean engineering. Two-dimensional (2-D) motions of the flexible cylinders towed underwater have been investigated by several researchers (Païdoussis, 1968; Dowling, 1988a, 1988b). Païdoussis (2004), (section 9, chapter 8) has reviewed the 2-D dynamics of towed cylinders systematically. Recently, Kheiri et al. (2013a, 2013b) explored the three-dimensional (3-D) dynamics of long pipes towed underwater. It is noted that the above mentioned studies

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on the dynamics of underwater towed systems are mostly focused on the linear aspect and a very limited number of studies have dealt with nonlinear models. A nonlinear model was developed for the dynamics of towed flexible slender cylinders by Kheiri et al. (2013c). The dynamic equations of a flexible slender body, which is connected at one end to a fixed support via a towrope and is free at the other end, are derived.

An important feature of the second type of axially moving systems is that the support ends are subjected to axial motion and hence the systems are of moving boundaries. In the above mentioned studies on the dynamics of underwater towed systems, the analytical model was developed in a floating coordinate system, which is attached to the towed systems, and then the dynamic equations were often derived in the floating coordinate system directly. However, it is noted that the Newton's law of motion holds only with respect to inertial frame of reference. As a result, the applicability of the model developed above needs to pay more attention. In addition, as to the mentioned studies on the dynamics of cylinder towed in fluid, the towing system is considered identical to a cylinder in axial flow (Païdoussis, 2004, pp. 930) if the cross-flow effect is neglected; and then the axial-direction added mass is equal to the lateral-direction virtual mass. However, as presented by Gosselin et al. (2007), an “axially added mass coefficient” should be introduced to better approximate the force of the surrounding fluid acting on the beam. Recently, with axially added mass coefficient taken into account, Ni et al. (2014) studied the linear vibration and stability of a cantilever beam attached to an axially moving base immersed in fluid, which belongs to the second type axially moving systems. In the work by Ni et al. (2014), two coordinate systems, i.e., the absolute coordinate frame which is an inertial frame of reference, and the moving coordinate frame attached to base, have been introduced. The dynamic equations were derived in the absolute coordinate frame firstly and then the equation of motion in the moving coordinate was obtained by utilizing the transformation of coordinates between the absolute and moving coordinates. The absolute coordinate frame is introduced to apply the Newton's law and the moving boundaries in the absolute coordinate frame become fixed in the moving coordinate frame. Thus, the modeling method presented in Ni et al. (2014) is of wide applicability and will be adopted in this study. It is well known that boundary conditions have significant influence on the vibration and stability of distributed parameter systems. For instance, the dynamics of cantilever slender structures and supported structures (both two ends are supported) may be very different (Païdoussis, 1998). The inextensibility condition of the cantilever slender structure cannot apply to the nonlinear dynamic analysis of slender structures with two supported ends since the axial stretch force should be considered. It is noted that the nonlinear model developed in Kheiri et al. (2013c) was the cantilever case and the axially added mass coefficient was not taken into account. Thus, the dynamics of a towed supported beam, especially the nonlinear aspects, with the consideration of axial added mass coefficient, need to be further studied. This motivates our study. The contributions of this paper mainly include following three points: (i) An nonlinear mathematical model for the underwater supported beams with axially moving supports with the consideration of nonlinear axial stretch induced by large deflections are developed; (ii) The pre-buckling and post-buckling dynamics of this system are studied carefully and various nonlinear dynamic motions including periodic-1, period-3, period-5, quasi-periodic and chaotic motions are detected; (iii) Although the numerical study in this paper focuses on the constant moving speed, the dynamic equations with the consideration of time-dependent moving speeds, which are derived in the problem formulation section, lay a solid foundation for further studies such as the dynamic response in acceleration process (Gosselin et al., 2007) and the parametric resonance vibration of this system which are induced by the fluctuation of moving speeds. In fact, parametric resonances have significant effects on the dynamics of slender structures, as can be seen in Öz and Pakdemirli (1999); Öz et al. (2001) and Panda and Kar (2008).

The rest of this paper is organized as follows. First, the equation of motion of the slender beam in fluid, which is towed by the axially moving supports, will be derived. Second, the discrete equation of motion is obtained by the Galerkin approach. Third, the linear dynamic analysis, which also applies to the linearized system of the nonlinear system of the trivial equilibrium configuration, will be given. The effect of axially moving speed of supports, mass ratio, and several other system parameters on the free vibration and stability of the beam is analyzed by calculating the natural frequencies and the lowest critical towing speed. Finally, the nonlinear dynamics of the system will be investigated. The buckled equilibrium solution will be obtained and its stability will be discussed in detail. Moreover, the pre-buckling and post-buckling dynamic responses will be studied.

2. Problem formulation

The system under consideration, shown in Fig. 1, consists of a beam towed by axially moving supports with known motion $L(t)$. Let this beam be of diameter D, with length l, area moment of inertia I, mass per unit length m and modulus of elasticity$E$. Consider this system to be immersed in an incompressible fluid of density ρ, with boundaries sufficiently distant to have negligible effect on the fluid forces on the beam. In the present study, it is assumed that no separation occurs in the flow around the beam, and that the forces of the fluid acting on a beam element are the same as those acting on a corresponding element of a long undeformed beam of the same cross-sectional area and inclination. Moreover, the effects of shear deformation and rotary inertia of the beam are neglected. In addition, the two ends of the beam is assumed to be simple supported and the gravity are neglected to simplify analysis.

Two coordinate systems will be introduced in this problem, the absolute coordinate frame (x, z) fixed in a certain spatial point and the moving coordinate frame $(\bar{x}, \bar{z})$ fixed in the left support ends. It is noted that the moving boundaries in the absolute coordinate system will become fixed in the moving coordinate system. The equation of motion in the absolute coordinate will be given firstly. Then, the equation of motion in the moving coordinate will be obtained by the coordinate transformation.

The axial and transverse displacement of beam are denoted as $u(x,t)$ and $w(x,t)$, respectively. Consider the balance law of an infinitesimal element of the beam, the equation of motions in the absolute coordinate system is obtained (Eqs. (17) and (18) in Ni et al., 2014)

$$(m+\beta M)\left(\frac{\partial}{\partial t}+\dot{L}\frac{\partial}{\partial x}\right)^2 u-\frac{\partial}{\partial x}(N+(pA))+\frac{1}{2}C_T\left(\frac{M}{D}\right)\dot{L}^2+\frac{\partial p}{\partial x}A=0 \tag{1}$$

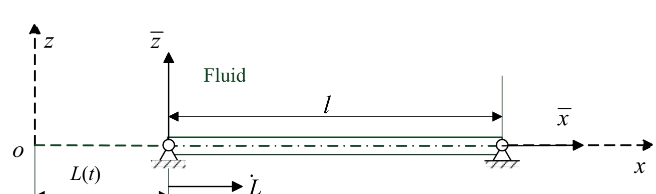

Fig. 1. The beam attached to axially moving supports immersed in fluid.