Ocean Engineering 108 (2015) 749–759

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

On the modelling of swell spectra

Cláudia Lucas, C. Guedes Soares *

Centre for Marine Technology and Ocean Engineering (CENTEC), Instituto Superior Técnico, Universidade de Lisboa, Portugal

ARTICLE INFO

Article history:
Received 8 November 2014
Accepted 13 August 2015
Available online 19 September 2015

Keywords:
JONSWAP spectra
Gaussian spectra
Swell spectra
Double peaked spectra

ABSTRACT

The ability of the JONSWAP model to describe swell spectra is analysed and the quality of the fit of this model is compared with the fits produced by a Gaussian model. The study is based on directional wave spectra from measurements made at the Maui location situated in the west coast of New Zealand and at the Bonga location situated near the coast of Africa. The analysis is performed for single peak swell sea states and for the swell component of two peaked spectra.

The comparison of the spectral parameters and the fit of the models with the measured narrow swell spectra, suggest that the JONSWAP model describes better the swell spectrum than the Gaussian model, in particular in the high frequency range.

1. Introduction

The energy distribution of a sea state can be described by a frequency spectrum, which can be estimated from a record of the sea surface elevation at a given point. This type of information is used to study the behaviour of ships, maritime and coastal structures subjected to the action of waves.

It has been shown that spectra of wind waves have a self-similar form depending on some parameters (Guedes Soares, 1984, 1998). The forcing of waves by wind generates non-linear interactions among waves, which induce the transfer of energy across energy bands leading to spectra with a shape that is the same for a given level of sea state development.

When the wind stops acting on a given sea area the forcing of the wave system disappears and it propagates almost without interaction among its components. This means that the lower frequency components will travel faster and thus, as the wave system travels away from the generation source, it will gradually lose its high frequency components which have a lower group velocity. Consequently, as the swell system propagates, and as the age of the sea state increases, the shape of the spectrum will change and become narrower with most of the energy concentrated around its peak.

The wind waves spectra were the first to be studied, and there are several formulations that describe it. After a period with several proposals of spectral shapes in the 1950s and 1960s, the spectrum of Pierson–Moskowitz (Pierson and Moskowitz, 1964) gained general acceptance of having the adequate shape to describe the spectra of fully developed sea states, representing thus balance between the forcing of the wind and wave conditions.

Later, the JONSWAP formulation (Hasselmann et al., 1973) was proposed to describe developing spectra, representing the non-stationary growth of the energy of waves as a result of wind forcing. During the development phase of the sea state there is a higher concentration of energy around the dominant frequency and the spectrum looks more peaked than the Pierson–Moskowitz one. This higher concentration of energy is represented in the formulation by a peak enhancement factor included in an expression that modifies the Pierson–Moskowitz spectrum.

Generally the waves that are generated by a very strong wind in a relatively short fetch have a high value of the peak enhancement factor. After this stage of growth of the energy spectrum under the action of the wind it will tend to become stationary or saturated when staying completely developed and then the peak enhancement factor tends to one and the JONSWAP spectrum will become the spectrum of Pierson–Moskowitz. In that sense it can be considered a generalisation of the first one (Guedes Soares, 1998).

It has been shown in numerous occasions that the wave climate in different sites results from the combination of a local system of wind waves, with a swell that was caused by a storm elsewhere and propagates already without effect of wind to the place of study (Guedes Soares, 1991; Lucas et al., 2011). Therefore it is important to be able to model properly those spectra.

Strekalov and Massel (1971) proposed one of the first models to describe double-peaked spectra, with one high frequency spectrum describing the wind driven component and a Gaussian shaped model describing the swell system. Another model was proposed by Ochi and Hubble (1976) by combining two Gamma spectra, which in practice represent the shape of the Pierson–Moskowitz spectrum. Guedes Soares (1984) proposed a model that represents both sea components by JONSWAP spectra of different

* Corresponding author.
E-mail address: c.guedes.soares@centec.tecnico.ulisboa.pt (C. Guedes Soares).

http://dx.doi.org/10.1016/j.oceaneng.2015.08.017
0029-8018/

peak frequencies. The choice to model the swell component by a JONSWAP spectrum was based on the fact that it is able to fit narrow spectra as one would expect from the swell component, as demonstrated by Goda (1983). This approach was also adopted by Torsethaugen (1993), who considered two JONSWAP spectra of varying peak enhancement factor, an option that provides better fit than the model of Guedes Soares (1984) but with increased number of spectral parameters to be estimated.

Although the discussion about modelling swell is an old one and appeared to have been settled, it was recently reopened by the need of the offshore industry to consider whether better models could be adopted (e.g Ewans, 2015) and among the various forms that have been under consideration (Olagnon et al., 2013) is the Gaussian model. This has motivated the present study that revisits the problem, comparing how the JONSWAP and the Gaussian model perform when fitting measured swell spectra.

This work starts by analysing measured spectra of a long period, corresponding to swell wave systems, and at a later stage considers two peaked spectra and analyses the shape of their swell component.

2. Spectral models

The objective of this work is to obtain a good spectral description of the swell spectrum and to find out which of the models, JONSWAP model or the Gaussian shaped model describes better the swell systems. The JONSWAP model depends on the peak frequency, f_p, and the peak enhancement factor, γ given by

$$S(f) = c\left(\frac{f}{f_p}\right)^{-5}\exp\left(-1.25\left(\frac{f}{f_p}\right)^{-4}\right)\gamma^{\exp\left(-(f-f_p)^2/2\sigma^2 f_p^2\right)} \quad (1a)$$

with

$$\sigma = 0.07 \quad \text{for} \quad f \le f_p \quad (1b)$$

$$\sigma = 0.09 \quad \text{for} \quad f > f_p \quad (1c)$$

The constant c can take different forms and the one that is adopted here depends on the significant wave height, H_s (Goda, 1979):

$$c = \frac{5H_s^2}{16f_p}\left(1.15 + 0.1688\gamma - \frac{0.925}{(1.909+\gamma)}\right)^{-1} \quad (1d)$$

The second model used and fitted to the data, is the Gaussian one, which is defined by the Gaussian probability function (Holthuijsen, 2007):

$$p(\eta) = \frac{1}{(2\pi m_0)^{1/2}}\exp\left(-\frac{\eta^2}{2m_0}\right) \quad (2)$$

For a zero-mean, $S(\eta)=0$, where $m_0^{1/2}$ is the standard deviation σ of the surface elevation η. The random sea-surface elevation can be treated as a stationary Gaussian process, so all its characteristics can be expressed in terms of the moments of that spectrum, defined as follows:

$$m_0 = \int_0^\infty f^n S(f)df \quad \text{for} \quad n = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \quad (3)$$

where m_n is the "nth-order moment" of $S(f)$. The zeroth-order moment is given by

$$m_0 = \int_0^\infty S(f)df \quad (4)$$

The variance of the surface elevation σ^2 is equal to the zeroth-order moment:

$$\sigma^2 = S\{\eta^2\} = \int_0^\infty S(f)df = m_0 \quad \text{for} \quad \mu_\eta = S\{\eta\} = 0 \quad (5)$$

The Gaussian distribution is symmetric having thus similar tails in the two sides, which allows it to have contributions for negative values of the frequency, at least in theory. This is a major difference from all models of wind wave spectra, which are asymmetric with much longer tails in the high frequency region and without tails in the low frequency range.

The theoretical spectrum JONSWAP is estimated by fitting to the measured spectrum using a non-linear least squares method to minimize the deviations between the theoretical and measured spectral ordinates. The variables used in the fit of the theoretical JONSWAP to the measured spectrum are the significant wave height, H_s, and the peak period, T_p, with $\gamma=3.3$ a fixed parameter.

The fitting procedure using the non-linear least squares method consists in an iterative optimisation process that finds the combination of H_s and T_p variables that minimises the residual or errors of fitting (i.e., sum of squares errors between the theoretical fit and measured spectra). The H_s and T_p values of the measured spectrum were used as initial condition for the optimisation problem.

For the second model, the Gauss model, determined by fitting a Gaussian curve (a normal distribution) to the measured spectra. The parameters of the Gaussian distribution are the mean and the standard deviation and were estimated using also a non-linear least squares method.

These two models will be applied to measured swell spectra of sea states that have only one wave system. They will then be applied to the swell component of double peaked spectra, from which they have been determined. A double peaked spectrum is assumed to be described by the sum of the two wave systems, the swell spectrum $S_s(f)$ and the wind sea spectrum $S_w(f)$:

$$S(f) = S_s(f) + S_w(f) \quad (6)$$

Therefore, starting from a measured two peak spectrum, an analysis is necessary to separate the two spectral components so that the study can be applied to the swell components.

For scalar spectra that are based only on the frequency, the method adopted by Guedes Soares (1984), with any of the improvements described in Guedes Soares and Nolasco (1992), or Rodriguez and Guedes Soares (1999) can be used. This method has been compared with others that take into account additional features of the spectra and was found to be more accurate (Ewans et al., 2006).

For directional spectra the adopted method of identifying and classifying multipeaked spectra is the one presented by Boukhanovsky et al. (2007) and Boukhanovsky and Guedes Soares (2009) and latter applied by Lucas et al. (2011) to various ocean locations. This method is able to classify the multipeaked directional spectra and the identification is based on the frequency and directional information. Thus, a set of general sea states can be identified, depending on the type of spectra. The input data of this method is the directional spectra, with the objective of isolate the spectra in which the swell wave system prevails and characterise it.

The classes in the spectra are *one-peaked spectra:* one wave system prevails, either the wind waves (class I) or the swell (class II), only one peak is pronounced; the *double-peaked spectra* (classes III, IV*):* two wave systems exist simultaneously. *Multi-peaked spectra*: complicated wave fields with two or more swells (class V). In this case, the spectrum has more than two pronounced peaks.

For two-peaked spectra two sub-classes are separated with respect to the wave generating conditions associated with the wave fetch and the duration of wave propagation, or the age of the system: the "matured" sea and the complex sea. The complex sea class (class IV) mostly consists of two wave systems, wind waves and swell, with the sub-classes wind waves and "fresh" swell and wind waves and "old" swell.

The "matured" sea class is described by double-peaked spectra with two swell systems (class III). Generally, it includes all other