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Experimental verification of a global exponential stable nonlinear wave encounter frequency estimator

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ABSTRACT

This paper presents a global exponential stability (GES) proof for a signal-based nonlinear wave encounter frequency estimator. The estimator under consideration is a second-order nonlinear observer designed to estimate the frequency of a sinusoid with unknown frequency, amplitude and phase. The GES proof extends previous results that only guarantee global $\mathcal{K}-$exponential stability. Typical applications are control and decision-support systems for marine craft, where it is important to know the sea state and wave frequency. The theoretical results are verified experimentally by analyzing data from towing tank experiments using a container ship scale model. The estimates for both regular and irregular waves confirm the results. Finally, the estimator is applied to full-scale data gathered from a container ship operating in the Atlantic Ocean during a storm. Again the theoretical results are confirmed.

1. Introduction

Estimation of the wave encounter frequency is an important part of sea-state prediction, which is of great importance in many marine operations as well as control systems design. Sea-state estimation also provides information to increase both the safety of operations at sea and the performance of control systems for ships affected by waves.

To increase the safety of operations, knowledge of the wave encounter frequency can be used for prediction of extreme waves, parametric roll resonance and in-service monitoring. Knowledge of the sea-state, and in particular the encounter frequency, is also important to increase the performance of marine control systems. For ship autopilot and dynamic positioning (DP) systems knowledge of the encounter frequency allows for better tuning of the low-pass and notch filters used in wave filtering (Fossen, 2011, Ch. 11). On-line adjustment of controller and observer gains also require the knowledge of the wave encounter frequency (Fossen and Strand, 1999). This allows for automatic gain scheduling of autopilots and DP systems.

1.1. Sea-state estimation

In the literature several techniques for estimation of the wave encounter frequency or wave spectra have been presented. The classical method is to obtain the wave spectrum from *Fast Fourier Transform* (FFT) frequency spectral analysis (Enshaei and Brimingham, 2012). Unfortunately, creating a FFT frequency spectrum takes time and consequently it results in back-dated information when estimating the time-varying wave encounter frequency. This is due to the moving window necessary for applying the FFT frequency spectral analysis. Hence, it is impossible to estimate a time-varying wave encounter frequency without lag.

More advanced spectral estimation techniques allow estimation of directional wave spectra (Nielsen, 2006). This can be done by parametric or non-parametric modeling. The parametric modeling approach typically assumes that the wave spectrum is parametrized such that it can be estimated using least-squares parameter matching of a bimodal spectrum for stationary vessels (Tannuri et al., 2003) and moving vessels (Nielsen, 2006). The non-parametric modeling or Bayesian approach uses stochastic processes to match the frequencies for stationary vessels (Iseki and Ohtsu, 2000) and moving vessels (Nielsen, 2006). These techniques have the same disadvantages with respect to acquisition times as the FFT frequency spectral analysis. However, besides the frequency of the waves, they also supply directional information.

Another approach to wave encounter frequency estimation is to estimate the peak frequency instead of the entire wave spectrum. This is a valid approach for the application of sea-state estimation when designing control systems, since the peak frequency of the spectrum is used for wave filtering (Fossen, 2011, Ch. 11). Approaches using Kalman

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filters can be found in Belleter et al. (2012) and Hassani et al. (2013). However, these approaches require a dynamic model of the vessel. A signal-based approach was developed in Belleter et al. (2013), where the measured roll or pitch angle is used to estimate the wave frequency. This approach is taken in this paper and experimental verification is included to justify the results.

1.2. Frequency estimation

Frequency estimation of oscillating signals is a well studied problem in the signal processing literature. A discrete-time algorithm for a multifrequency signal based on an adaptive notch filtering was first proposed by Regalia (1991). A continuous-time version of this algorithm was presented in Bodson and Douglas (1997), while Hsu et al. (1999) have derived a globally convergent continuous-time frequency estimator for a single frequency signal.

An adaptive technique based on the persistency of excitation (PE) of oscillating signals was proposed in Marino and Tomei (2002), and extended by Xia (2002) and Hou (2012). Two discrete-time algorithms based on PE can also be found in Stotsky (2012).

The approach taken by the authors is based on the internal model principle for identification of a single frequency. This was first introduced in Nikiforov (1997) and further extended by Aranovskiy et al. (2007), Bobtsov (2008), and Aranovskiy and Bobtsov (2012).

1.3. Main contribution

The main result of the paper is a nonlinear signal-based wave encounter frequency estimator, which effectively estimates the ship wave encounter frequency from heave, pitch or roll motion measurements. The wave encounter frequency estimator under consideration is designed to estimate the frequency of a sinusoid with unknown frequency, amplitude and phase by modifying the algorithm of Aranovskiy et al. (2007) to include an adaptive gain-switching mechanism. The GES proof extends previous results (Belleter et al., 2013) that only guarantee uniform global asymptotic stability (UGAS) and uniform local exponential stability (ULES). This is also referred to as global $\mathcal{K}-$exponential stability (Sørdalen and Egeland, 1995).

The main motivation for introducing a gain-switching mechanism is that it is important to improve the convergence of the estimator in situations with little excitation (e.g. small roll and pitch angles) and vice versa. Typical applications are marine craft control and decision-support systems where it is important to know the sea state and wave frequency. The wave encounter frequency estimator is experimentally verified through towing tank tests in both regular and irregular waves. The estimator is also verified for 9-hours of data gathered onboard the container vessel *Clara Maersk* during a storm across the North Atlantic Ocean.

The wave estimator is implemented in real-time and consequently it is much faster than the real-time requirement of the ship autopilot and DP control systems, which typically samples data at 1–10 Hz. FFT is an off-line algorithm, which use batches of data (moving window). The computational footprint is higher and significantly affected by the acquisition time and numerical processing of the data.

1.4. Organization of the paper

The paper is organized as follows: In Section 2 the wave encounter frequency estimation problem is introduced and the Aranovskiy fixed-gain frequency estimator is reviewed. Section 3 presents the switching-gain frequency estimator and GES of the equilibrium point of the estimation error dynamics is proven. Section 4 contains experimental verification using towing tank experiments and full-scale data of a container ship.

2. Estimation of the wave spectrum encounter frequency

Characterization of the sea state for marine operations is generally done in terms of a limited number of fundamental parameters, which are used to calculate approximations of the wave spectrum. Those parameters are the significant wave height H_s, the wave modal frequency (peak frequency) ω_0, and the wave encounter angle β_e that is the relative angle between the vessel heading and the main direction of the wave train. Knowledge of those parameters may reveal to be of extreme importance in order to schedule and perform activities at sea in a safe, reliable and cost effective manner.

For vessels in transit at forward speed $U>0$ the experienced wave excitation does not occur at the modal frequency ω_0 because of the *Doppler shift*. The frequency observed from the vessel in motion is given by

$$\omega_e(\omega_0, U, \beta_e) = \left|\omega_0 - \frac{\omega_0^2}{g} U \cos(\beta_e)\right| \tag{1}$$

which is known as the wave encounter frequency. Awareness about ω_e would allow performance enhancement of ship control systems. For instance autopilots and DP systems use wave filters, which are tuned to suppress oscillations at the encounter frequency, in order to reduce the workload of the steering and propulsion systems.

Although waves are usually described as narrow-band stochastic processes, the associated spectrum is certainly richer in frequency content than a single sinusoid. Nevertheless spectral analysis of wave-induced vessel motions usually displays a dominant frequency associated with the peak of the spectrum. During the transient the natural frequencies of the different modes can be observed in the spectrum giving rise to multiple peaks. However, if the waves are large enough the ship will oscillate at ω_e in all 6 degrees-of-freedom in steady state. For multi-peaked wave spectra with a dominant peak the proposed method will provide an estimate close to the frequency of the highest peak. Analytically the problem can be formulated as

Problem definition (Wave encounter frequency estimation) *Given the signal in the form*:

$$y(t) = A(t)\sin(\omega_e t + \epsilon) \tag{2}$$

with $A(t)$ the unknown amplitude, ω_e the unknown frequency and ϵ the unknown phase, reconstruct on-line the frequency ω_e based solely on noisy measurements of $y(t)$.

2.1. The Aranovskiy fixed-gain frequency estimator

Before presenting the main contribution of the paper (Theorem 1), we first review the signal-based frequency estimator proposed by Aranovskiy et al. (2007), which is instrumental in our design.

The sinusoidal signal (2) can be represented by the differential equation:

$$\ddot{y} = \varphi y \tag{3}$$

where $\varphi := -\omega_e^2$ is treated as an unknown parameter. The frequency ω_e of the signal (2) can be estimated using an auxiliary filter (Aranovskiy and Bobtsov, 2012):

$$\dot{\zeta}_1 = \zeta_2 \tag{4}$$

$$\dot{\zeta}_2 = -2\omega_f \zeta_2 - \omega_f^2 \zeta_1 + \omega_f^2 y \tag{5}$$

where the filter cut-off frequency must be chosen such that $0 < \omega_e < \omega_f$. The transfer function corresponding to (4) and (5) is found by Laplace