Ocean Engineering 96 (2015) 21–33

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

Flow-induced vibrations of four circular cylinders with square arrangement at low Reynolds numbers

Zhaolong Han [a,g], Dai Zhou [a,b,c,*], Tao He [a,d], Jiahuang Tu [a], Chunxiang Li [e], Kenny C.S. Kwok [f], Congqi Fang [a]

[a] School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai, China
[b] State Key Laboratory of Ocean Engineering, Shanghai Jiao Tong University, Shanghai, China
[c] Institute of Oceanology, Shanghai Jiao Tong University, Shanghai, China
[d] School of Civil Engineering, University of Birmingham, Birmingham, United Kingdom
[e] Department of Civil Engineering, Shanghai University, Shanghai, China
[f] Institute for Infrastructure Engineering, University of Western Sydney, Penrith, NSW, Australia
[g] Cullen College of Engineering, University of Houston, Houston, TX, USA

ARTICLE INFO

Article history:
Received 15 January 2014
Accepted 3 December 2014

Keywords:
Arbitrary Lagrangian–Eulerian
Characteristic-based split finite element method
Four cylinders
Square arrangement
Flow-induced vibration

ABSTRACT

Flow-induced vibrations (FIV) of four identical circular cylinders placed in a square arrangement are numerically investigated. Modeled as a spring-damping system subjected to uniform flows, each cylinder is allowed to freely oscillate with equal natural frequencies in the inline and transverse directions. The spacing ratio, L/D, remains 5, where L is the central distance of any two adjacent cylinders and D the cylinder diameter. The Reynolds numbers are chosen as $Re=80$ and 160. The incidence angle of the incoming uniform flow is $\alpha=0^{\circ}$. The mass ratio for each cylinder is $M_r=6.0$ and the reduce velocity, U_r, varies from 3 to 14. The coupled system is numerically resolved by a semi-implicit characteristics-based split (CBS) finite element algorithm under the arbitrary Lagrangian–Eulerian description. The calculated results are analyzed in detail. In particular, some intrinsic mechanisms are interpreted on the cylinder responses and the wake patterns. The unsymmetrical figures of "8" and "O", and other irregular figures are observed in the cylinders' X–Y trajectories. Besides the "4S" wake pattern, the "2P+2S" pattern is discovered herein. The "dual-resonance" phenomenon, which indicates the cylinders' synchronizations occurring in both the inline and transverse directions, is detected in this work.

1. Introduction

Over the past decades, flow-induced oscillations of the multi-cylinder system have gained growing interests from both the scientific and practical communities. Studies on such a topic are of great significance for a variety of engineering structures, such as cables, bridges, marine risers and ocean platforms. The knowledge can also reveal important flow features in the vortex-induced vibration (VIV) dynamics. When long and flexible cylinders are immersed in the fluid field, they will be excited to vibrate by the external forces from the flow that passes by. Meanwhile, the movements of the cylinders will in return change the flow performance and characteristics.

When there is only one cylinder freely oscillating in the fluid domain, commonly known flow patterns such as the "2P", "2S" and "P+S" ("P" here means a pair of vortices, and "S" means one) can be found for different parameter settings. Meanwhile, the "lock-in" or "lock-on" performance may appear which means that as the vortex shedding frequency approaches its natural frequency, the cylinder will experience a vibration with relatively large amplitude. Moreover, a "Fig. 8" is often observed in the X–Y trajectory of the cylinder movement. For more details of this one-cylinder subject, review work by Williamson and Govardhan (2004), Sarpkaya (2004), Gabbai and Benaroya (2005), and Bearman (2011) can be referred to.

If there are two identical cylinders in the fluid field that vibrate together, the results are expected to be more complicated. This scenario not only involves the VIV caused by the incoming uniform flow, but also the effect of the wake flow shedding from the

* Correspondence to: School of Naval Architecture, Ocean and Civil Engineering, State Key Laboratory of Ocean Engineering, Shanghai Jiao Tong University, No. 800 Dongchuan Road, Minhang District, Shanghai 200240, China.
Tel.: +86 21 34206195; fax: +86 21 34206198.
E-mail addresses: han.arkey@gmail.com (Z. Han), zhoudai@sjtu.edu.cn (D. Zhou).

http://dx.doi.org/10.1016/j.oceaneng.2014.12.002
0029-8018/

upstream cylinder on the downstream one. This subject is also important but not investigated as extensively as the one-cylinder issue. Due to the interference effect of the two cylinders, the fluid dynamics characteristics vary a bit from the one-cylinder results. Using experimental and numerical approaches, valuable studies have been conducted; see Allen and Henning (2003), Assi et al. (2006), Brika and Laneville (1999), Carmo et al. (2011), Mittal and Kumar (2001), Papaioannou et al. (2008), Prasanth and Mittal (2009a,b), Bao et al. (2011, 2010a).

The four-cylinder array is also commonly used in the engineering applications, such as in marine risers and in ocean platform systems. If the VIV characteristics were not well understood or if the structures were not well designed, they might be damaged by the long-term external forces from the environmental water or wind. Hence, understanding the features of flow patterns and the fluid–structure coupling dynamics for the four-cylinder system is of great significance in the engineering applications.

Regarding the investigation approaches, the experiments and numerical simulations are commonly used. In most experimental work, the Reynolds numbers are in the turbulence flow regime, and should be changed with the flow inlet velocities. On the contrary, the numerical simulations can easily realize the changing of a variety of parameters such as Reynolds number and reduced velocity. Meanwhile, the simulated results can also reveal important features of the VIV dynamics observed in the experiments. Hence, in our opinion, the combination and comparison of the experimental investigation and numerical simulation can provide deep insight into the VIV system.

There have been a number of investigations on flow past four cylinders. For instance, Lam and So (2003) Lam et al. (2003a, 2003b, 2008), Lin et al. (2008) have adopted experimental approaches and numerical calculations to understand the patterns for the flow past four cylinders in a square configuration at different Reynolds numbers and incidence angles. Their publications have demonstrated that the flow characteristics and patterns are highly determined by the Reynolds number and spacing ratios, L/D (L is the center-to-center distance and D the cylinder diameter). In our previous work, a numerical study was also conducted on flow over a stationary four-cylinder array using spectral element method (Han et al., 2013a). The wake patterns were categorized and the hydrodynamics coefficients of the cylinders were presented and analyzed.

However, the four cylinders in the work mentioned above were only assumed to be stationary. According to the authors' knowledge, there are still inadequate publications regarding the VIVs of a four-cylinder array. Zhao and Cheng (2012) conducted a 2-dimensional numerical investigation on 2-DOF VIV of four rigidly-coupled circular cylinders in a square configuration using a finite element scheme of Reynolds-averaged Navier–Stokes (RANS) equations and SST k–ω turbulence model. The results showed that the flow incidence angle has a great impact on the cylinders' responses. Sanaati and Kato (2013) performed experimental investigation on the 3-D VIVs of flexible four-cylinder arrays, showing that the cylinders were excited up to the second and fourth mode of vibrations for the cross-flow and inline directions, respectively.

In this study, the subject of flow-induced vibrations of a four-cylinder array is factored in. Slightly different from Zhao and Cheng (2012)'s work, the Reynolds number is set as low as possible while still remaining within the laminar flow regime. Meanwhile, each circular cylinder is identically associated with a 2-DOF (inline and transverse) movement and able to vibrate freely. Distance between two adjacent cylinders' centers is defined as 5D, which is expected to allow the mature vortices to be fully generated. This also allows their shedding down from the upstream cylinders so that the wake-induced vibrations can be observed. Low Reynolds numbers, $Re=80$ and 160 (which are within the laminar flow regime), are adopted enabling a 2-dimensional (2-D) study. The purpose is to reveal the basic mechanisms of fluid–solid interaction, and to investigate the effect of the reduced velocities varying from 3 to 14 for each cylinder. Further, some experimental data are compared with the numerical results to demonstrate the basic characteristics of the four-cylinder VIV system.

The structure of this paper is organized as follows. In Section 2, the governing equations and algorithms for the incompressible viscous flow and cylinder movements are introduced. In Section 3, the details of the problem are described and the numerical results for the flow-induced vibrations of the four-cylinder array with various reduced velocities are presented and discussed. Finally, main conclusions of the current work are summarized in Section 4.

2. Methods

2.1. Fluid governing equations

To describe the VIV of the four-cylinder system, the non-dimensional Navier–Stokes (N–S) equation and the continuity equation for the incompressible viscous flow under the Arbitrary Lagrangian–Eulerian (ALE) framework, which was proposed by Donea et al. (1982), are adopted and shown in the following equations:

$$\frac{\partial u_i}{\partial t}+(u_j-w_j)\frac{\partial u_i}{\partial x_j}=-\frac{\partial p}{\partial x_i}+\frac{1}{\mathrm{Re}}\frac{\partial^2 u_i}{\partial x_j \partial x_j}, \tag{1}$$

$$\frac{\partial u_i}{\partial x_i}=0, \tag{2}$$

where u_i is the fluid velocity, and w_i the grid velocity tensor in the ith direction, p represents the pressure, x_i ($i=1,2$ in 2- and 1,2,3 in 3-D problems) the Cartesian coordinates, and t the time. The Reynolds number is defined as $Re=U_\infty D/\nu$, where $D=1$ is the characteristic length in the non-dimensional fluid domain. The U_∞ is the free stream velocity, ν the fluid kinematic viscosity.

These equations can be solved by many numerical discretization schemes, and here the solutions are obtained by the characteristic-based split (CBS) finite element method (Zienkiewicz and Codina, 1995). There are other similar schemes such as the Streamline Upwind Petrov–Galerkin (SUPG) method (Brooks and Hughes, 1982), Galerkin least square (GLS) method (Hughes et al., 1989), the Taylor–Galerkin (TG) method (Donea, 1984), and the two-step Taylor-characteristic-based Galerkin (TCBG) method (Bao et al., 2010b). The CBS scheme is adopted because of its simple formulations and easy implementation in the computational code. Most of the CBS is constructed using an explicit form (Nithiarasu et al., 2006; Zienkiewicz et al., 1999). In the present work, the diffusion term and the stabilized term in the CBS method are semi-implicitly treated, expecting to provide a more stable feature than the explicit one (Han et al., 2013b, 2014a,b).

Under the framework of the fractional step method, the necessary steps in the temporal discretization are shown as follows:

$$\frac{u_i^*-u_i^n}{\Delta t}-\frac{1}{2Re}\frac{\partial^2 u_i^*}{\partial x_j \partial x_j}-\frac{\Delta t}{2}(u_k^n-w_k^n)\frac{\partial}{\partial x_k}\left(\left(u_j^n-w_j^n\right)\frac{\partial u_i^*}{\partial x_j}\right)= -\left(u_j^n-w_j^n\right)\frac{\partial u_i^n}{\partial x_j}+\frac{1}{2Re}\frac{\partial^2 u_i^n}{\partial x_j \partial x_j}, \tag{3}$$