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Wavelet analysis of non-stationary sea waves during Hurricane Camille

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ABSTRACT

The wavelet transform is applied to wave records, containing abnormal waves. The field data are from hurricane Camille. The abnormal waves are readily identified from the wavelet spectrum as an area of high energy, well localized in time and frequency. The wavelet spectrum of abnormal waves, appearing in a group of high waves has maximum value at the time of abnormal wave occurrence and it is concentrated in a narrow frequency range. In contrast the single abnormal wave has wavelet spectrum covering a wide frequency range. The vertical asymmetry of single abnormal wave is reflected in asymmetrical distribution of wavelet transforms. The presence of components with similar phases as a factor for abnormal wave occurrence is studied. Before and after wave crest of asymmetrical abnormal wave, the phases are more dispersive than that during the abnormal crest occurrence.

1. Introduction

Abnormal waves, called also freak or rogue waves are extremely large and usually very steep waves. They are transient events and thus, essentially non-stationary ones. The term abnormal wave is given to a wave which is larger than the ones that would normally be expected by certain wave theory. Based on the 20 min duration of wave record, Dean (1990) has suggested that a wave should be classified as abnormal if its height is greater than twice the significant wave height. There are various other criteria for the identification of abnormal wave and reviews of them can be found in Clauss (2000), Guedes Soares et al. (2003) and Kharif et al. (2009).

Abnormal waves have been registered in the ocean: in the North Sea (Guedes Soares et al., 2003; Cherneva and Guedes Soares, 2008, 2014; Veltcheva and Guedes Soares, 2007), the Sea of Japan (Yasuda and Mori, 1997; Liu and Mori, 2000), the Gulf of Mexico (Guedes Soares et al., 2004; Veltcheva and Guedes Soares, 2012), the Pacific Ocean (Chien et al., 2002) and in the Baltic Sea (Didenkova, 2011). These field data are important source of information about characteristics of abnormal waves, occurring in real sea conditions.

To understand the nature of abnormal wave from a full-scale wave data record however depends to great extent on the applied method of analysis. The abnormal wave is a non-stationary transient event and its study by conventional methods for wave data analysis is difficult due to the assumption of stationarity of the analyzed process in those methods. The Fourier spectral analysis, which also relies on stationarity, provides information representative of the global properties of data time series. The characteristics of ocean waves, determined from the Fourier spectrum are integral ones and they do not give any information about the local time variation of energy. The assumption of stationarity precludes the detection of transient events in the data time series.

An alternative to evaluate the local variations of energy within the time series is provided by time–frequency distribution of energy. There are different methods for estimation the variations of energy simultaneously in time and frequency domain such as short-time Fourier transform or spectrogram (Guedes Soares and Cherneva, 2005), generalized time–frequency spectrum, bispectrum and trispectrum, (Cherneva and Guedes Soares, 2008), the Wigner spectra (Cherneva and Guedes Soares, 2011, 2014) and the Hilbert–Huang Transform (Huang et al., 1999; Schlurmann et al., 2000; Veltcheva, 2002; Veltcheva and Guedes Soares, 2004, 2007, 2012).

Wavelet transform (WT) is the another method, used for studying the time–frequency distribution of energy. Wavelet transform decomposes the data time series into wavelets, which are scaled and shifted version of preliminary chosen mother wavelet. The wavelet analysis yields localized time–frequency distribution of energy by multiplying the signal with the wavelets. The wavelets have different width as the transform is computed for every single spectral component. In contrast to the Fourier transform, the wavelet transform allows localization of energy both in frequency and time domains. For this reason WT is suitable for analysis of non-stationary processes and it has been used previously for analysis of non-stationary wave data by Massel (2001), Huang (2004), Liu (2000) among others. Liu and Mori (2000), Schlurmann et al. (2000) and Ewans and Buchner (2008) have applied wavelet transform for analysis of abnormal wave data.

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The ability to gain insight in the time variation of spectral components is important for the studying of abnormal waves. One of the mechanisms responsible for abnormal wave formation is a linear superposition of harmonic components with coherent phases. Thus the presence of wave components with similar phases is an important factor for abnormal wave occurrence. Another mechanism concerns the generation of abnormal waves as a result of the modulation instability. The wave modes are interacting to redistribute the spectral energy and during this rapid shifting of wave energy in the spectrum the occurrence of large waves significantly increases. The three-dimensional numerical simulations by higher order nonlinear Schrodinger equation, carried by Socquet-Juglard et al. (2005) showed that the occurrence of abnormal waves is correlated with the spectral change for the case of long-crested waves. The long-wave flume experiments of Onorato et al. (2004) also demonstrated an increase in the density of abnormal waves associated with spectral instability.

This work proposes the wavelet analysis for studying the time–frequency distribution of energy of wave records, containing abnormal waves. The wavelet transform is briefly introduced in Section 2. The application of wavelet transform to full-scale wave data is presented in Section 3. It is demonstrated that wavelet transform allows the description of time variation of local energy during the abnormal wave occurrence. The time–frequency distribution of wavelet transform and phase spectrum around the time of the abnormal wave occurrence shows that the decomposition of symmetrical abnormal wave in a group differs from the decomposition of single asymmetrical abnormal wave.

2. Wavelet transform

The major task of signal processing is to extract information about the phenomenon from the data record comparing the data signal with a set of known functions. This comparison is performed mathematically through the inner product of signal $x(t)$ and functions $\psi_n(t)$

$$c_n = \langle x, \psi_n \rangle = \int_{-\infty}^{\infty} x(t)\psi_n^*(t)dt \tag{1}$$

for a continuous signal. Here the asterisk denotes the complex conjugate. The inner product is a measure of similarity between the signal $x(t)$ and the function $\psi_n(t)$. The more the signal $x(t)$ is similar to the function $\psi_n(t)$ the bigger the inner product. The inner product (1) is called analysis or transform.

The original signal can be recovered if the basis $\{\psi_n(t)\}$ is orthonormal.

$$x(t) = \int_{-\infty}^{\infty} c_n \hat{\psi}_n \, dt \tag{2}$$

where $\{\hat{\psi}_n\}$ are orthonormal functions for the $\{\psi_n\}$ such as their inner product is $\langle \psi_n, \hat{\psi}_m \rangle = \delta_{mn}$, where δ_{mn} is the Kronecker delta function.

The reconstruction (2) is called synthesis or inverse transform or decomposition. The functions $\{\psi_n(t)\}$ form the basis of decomposition and there are many ways to decompose a signal. The basis of Fourier decomposition is set of harmonic functions with constant amplitudes and frequencies.

Wavelet transform proposed another way to decompose the signal into its constituent components. WT of signal $x(t)$ is defined by inner product

$$WT(\tau, b) = \langle x(t), g_{\tau b}(t) \rangle = \int_{-\infty}^{\infty} x(t) g_{\tau b}^*(t; \tau, b)dt \tag{3}$$

where the asterisk denotes the complex conjugate and $g_{\tau b}(t)$ is a family of continuously translated and dilated wavelets, generated from mother wavelet $g(t)$ by

$$g_{\tau b}(t; \tau, b) = \frac{1}{\sqrt{b}} g\left(\frac{t-\tau}{b}\right) \tag{4}$$

Here τ is the translation parameter, corresponding to the position of the wavelet as it is shifted through the signal; and b is the scale dilation parameter determining the width of the wavelet. The scale $b > 1$ stretches out or dilates the wavelet whereas scale $b < 1$ compresses the wavelet.

The wavelet coefficients $WT(\tau,b)$ is a measure of the correlation between wavelet and local segment of the signal. A large value of $WT(\tau,b)$ indicates that the signal has a major component of the frequency corresponding to the given scale. By varying the scale b and translating along the localized time τ, a time and scale variation of amplitude can be constructed.

A function $g(t)$ must satisfy the following basic properties in order to be mother wavelet:

1. The amplitude $|g(t)|$ must decay rapidly to zero when $|t| \to \infty$ and this condition ensures the localization of wavelet in time. The wavelet $g_{\tau b}$ has insignificant effect at time $|t| > \tau_{cr}$, where τ_{cr} is a critical time lag.
2. The wavelet must have zero mean. This condition ensures that the original signal can be obtained from wavelet transform coefficients $WT(\tau,b)$ by inverse wavelet transform as
$$x(t) = \frac{1}{C} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} WT(\tau, b) b^{-2} g_{\tau b}(t; \tau, b) d\tau \, db \tag{5}$$
in which
$$C = \int_{-\infty}^{\infty} \left(\omega^{-1} |G(\omega)|^2 \right) d\omega < \infty \tag{6}$$
where $G(\omega)$ is Fourier transform of function $g(t)$. This condition is known also as admissibility condition. It allows the reconstruction of a signal $x(t)$ from its continuous wavelet transform. In general the reconstruction of the original signal from wavelet components is valid for the orthonormal wavelets.
3. The wavelets are regular functions such as $G(\omega < 0) = 0$

There are many candidates for mother wavelets. As far as the wavelet transform should reflect the type of features, presented in the time series, the choice of the wavelet varies according to the specific application. Once the basic wavelet is selected, one will have to use it to analyze all the data. For wavelet analysis of wave data, especially appropriate is the Morlet wavelet due to its oscillatory nature and nearly identity of the Morlet scale parameter and the Fourier period.

The complex Morlet wavelet

$$g(t) = exp\left(-\frac{1}{2}t^2\right) \exp(ict) \tag{7}$$

is plane wave with frequency c, modulated by a Gaussian envelope of the unit width. The family of wavelets with Morlet mother wavelet is defined as

$$g_{\tau b}(t) = \frac{1}{\sqrt{b}} exp\left(-\frac{1}{2}\left(\frac{t-\tau}{b}\right)^2\right) \exp\left(ic\frac{t-\tau}{b}\right) \tag{8}$$

The Morlet wavelet was used by Liu (2000), Liu and Mori (2000), Massel (2001), Chien et al. (2002) and Huang (2004) for analysis of ocean wave data.

The wavelet analysis is performed in similar way to the short-time Fourier transform analysis, but the window width is changed as the transform is computed for every spectral component. The