Precision Engineering 38 (2014) 18–27

Contents lists available at ScienceDirect

Precision Engineering

journal homepage: www.elsevier.com/locate/precision

Tool life prediction using Bayesian updating. Part 2: Turning tool life using a Markov Chain Monte Carlo approach

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ARTICLE INFO

Article history:
Received 7 November 2011
Received in revised form 25 June 2013
Accepted 27 June 2013
Available online 22 July 2013

Keywords:
Tool wear
Taylor tool life
Bayesian updating
Discrete grid
Markov Chain Monte Carlo
Uncertainty

ABSTRACT

According to the Taylor tool life equation, tool life reduces with increasing cutting speed following a power law. Additional factors can also be added, such as the feed rate, in Taylor-type models. Although these models are posed as deterministic equations, there is inherent uncertainty in the empirical constants and tool life is generally considered a stochastic process. In this work, Bayesian inference is applied to estimate model constants for both milling and turning operations while considering uncertainty.

In Part 1 of the paper, a Taylor tool life model for milling that uses an exponent, n, and a constant, C, is developed. Bayesian inference is applied to estimate the two model constants using a discrete grid method. Tool wear tests are performed using an uncoated carbide tool and 1018 steel work material. Test results are used to update initial beliefs about the constants and the updated beliefs are then used to predict tool life using a probability density function. In Part 2, an extended form of the Taylor tool life equation is implemented that includes the dependence on both cutting speed and feed for a turning operation. The dependence on cutting speed is quantified by an exponent, p, and the dependence on feed by an exponent, q; the model also includes a constant, C. Bayesian inference is applied to estimate these constants using the Metropolis–Hastings algorithm of the Markov Chain Monte Carlo (MCMC) approach. Turning tests are performed using a carbide tool and MS309 steel work material. The test results are again used to update initial beliefs about the Taylor tool life constants and the updated beliefs are used to predict tool life via a probability density function.

1. Introduction

Tool wear can impose a significant limitation on machining processes, particularly for hard-to-machine materials such as titanium and nickel-based superalloys. Taylor first defined an empirical relationship between tool life and cutting speed using a power law [1]:

$$VT^n = C \quad (1)$$

where V is the cutting speed in m/min, T is the tool life in minutes, and n and C are constants which depend on the tool–workpiece combination. The constant C is defined as the cutting speed required to obtain a tool life of 1 min. Tool life is typically defined as the time required to reach a predetermined flank wear width (FWW), although other wear features (such as crater depth) may also be applied depending on the nature of the tool wear. The Taylor tool life equation can be extended to include other effects, such as feed rate [2]:

$$V^p f_r^q T = C \quad (2)$$

where f_r is the feed in mm/rev in turning and C, p, and q are constants which depend on the tool–workpiece combination. Note that in the extended Taylor tool life equation shown in Eq. (2), the constant C is dimensionless. The Taylor-type tool life model shown in Eq. (2) is deterministic in nature, but uncertainty exists due to: (1) factors that are unknown or not included in the model; and (2) tool-to-tool performance variation. For these reasons, tool wear is often considered to be a stochastic and complex process and, therefore, difficult to predict.

Previous efforts to model tool wear as a stochastic process are available in the literature [3–5]. Vagnorius et al. calculated the optimal tool replacement time by determining the probability of the tool failing before the selected time using a tool reliability function [3]. Liu and Makis derived a recursive formula to determine the cutting tool reliability. The maximum likelihood method was used to determine the unknown parameters in the reliability function

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http://dx.doi.org/10.1016/j.precisioneng.2013.06.007

[4]. Wiklund applied the Bayesian approach to monitor tool wear using in-process information [5]. The method presented in this paper uses Bayesian inference to predict tool life at the process planning stage. The distribution of the Taylor tool life constants, p, q, and C, are updated using experimental tool life results. The updated distributions of p, q, and C can then be used to predict tool life. The objective of the paper is to demonstrate the application of Bayesian updating to tool life prediction. The Taylor tool life model is used in this study, despite its potential limitations, because it is well-known and generally understood in the manufacturing community. Without loss of generality, the Bayesian updating method demonstrated in this paper can be applied to other available models [6].

2. Bayesian inference

Bayesian inference, which forms a normative and rational method for belief updating is applied in this work [7]. Let the prior distribution about an uncertain event, A, at a state of information, &, be $\{A|\&\}$, the likelihood of obtaining an experimental result B given that event A occurred be $\{B|A,\&\}$, and the probability of receiving experimental result B (without knowing A has occurred) be $\{B|\&\}$. Bayes' rule is used to determine the posterior belief about event A after observing the experiment results, $\{A|B,\&\}$ as shown in Eq. (3). Using Bayes' rule, information gained through experimentation can be combined with the prior prediction about the event to obtain a posterior distribution.

$$\{A|B,\&\} = \frac{\{A|\&\}\{B|A,\&\}}{\{B|\&\}} \tag{3}$$

As seen in Eq. (2), the Taylor-type tool life model assigns a deterministic value to tool life for the selected cutting speed and feed rate values. In contrast, Bayesian inference assigns a probability distribution to the tool life value at a particular cutting speed/feed rate combination. From a Bayesian standpoint, a variable which is uncertain is treated as a variable which is random and characterized by a probability distribution. The prior, or initial belief of the user, can be based on theoretical considerations, expert opinions, past experience, or data reported in the literature; the prior should be chosen to be as informative as possible. The prior is represented as a probability distribution and, using Bayes' theorem, the probability distribution is updated when new information becomes available (from experiments, for example). As a result, Bayesian inference enables a model to incorporate uncertainty in terms of a probability distribution and beliefs about this distribution to be updated based on experimental results.

In the Taylor-type tool life model provided in Eq. (2), there is uncertainty in the exponents, p and q, and in the constant, C. As a result, there is uncertainty in the tool life, T. This uncertainty can be represented as a joint probability distribution for C, p, and q and, therefore, for the tool life, T. Bayes' rule (Eq. (3)) can be used to update the prior joint distribution of C, p, and q using new information. The new distribution can then be used to update the distribution of tool life, T. In this case, the prior distribution $\{A|\&\}$ is the initial belief about constants C, p, and q. The updating of the constants can be completed using experimental data of tool life. For this case, Bayes' rule is:

$$\{p, q, C|T, \&\} \propto \{p, q, C|\&\}\{T|p, q, C, \&\} \tag{4}$$

where $\{p, q, C|\&\}$ is the prior joint distribution of p, q, and C, $\{T|p, q, C,\&\}$ is the likelihood of observing experimental result of tool life, T, given C, p, and q, and $\{p, q, C|T,\&\}$ is the posterior joint distribution of C, p, and q given an experimental result of tool life, T. Note that the denominator in Eq. (3), $\{B|\&\}$, acts as a normalizing constant. It is not included in Eq. (4).

According to Bayes' rule, the posterior distribution is proportional to the product of the prior and the likelihood. The prior is a three-dimensional joint distribution of the constants C, p, and q. The likelihood and, subsequently, the posterior are also three-dimensional joint distributions of C, p, and q. The grid-based method (see Part 1 of this paper) is computationally expensive for updating a joint distribution with three or more dimensions since it is dependent on the size of the grid. For example, a joint probability density function (pdf) of three variables with a grid size equal to 300 would require at least 2.7×10^6 computations for each update in the grid-based method. As an alternative, the Markov Chain Monte Carlo (MCMC) technique can be used to sample from multivariate posterior distributions for Bayesian inference [7]. Using the MCMC technique, samples can be drawn from the posterior multivariate distribution which can then be used to characterize the distribution. The single-component Metropolis–Hastings (MH) algorithm facilitates sampling from multivariate distributions without sensitivity to the number of variables [9,10]. The algorithm proceeds by considering a single variable at a time and sampling from a univariate proposal distribution. In this study, the single-component MH algorithm of the MCMC technique is used to sample from the joint posterior distribution of the constants C, p, and q. The remainder of the paper is organized as follows. Section 3 describes the use of the MH algorithm to sample from a univariate bimodal pdf and the application to Bayesian inference. Section 4 describes sampling from the joint posterior distribution using the single component MH algorithm. Tool life prediction using the posterior or the updated distributions of tool life is shown in Section 5. Section 6 compares the Bayesian approach to classical regression. Finally, the influence of prior and likelihood uncertainty is discussed in Section 7.

3. Markov chain Monte Carlo (MCMC) method

The Markov Chain Monte Carlo (MCMC) method is a sampling technique used to draw samples from a pdf. Samples are generated from the state space of the variable of interest using a Markov chain mechanism [8]. The most popular method for MCMC is the MH algorithm [9,10]. Let x be the variable of interest. The pdf of variable x is referred to as the target distribution and is denoted by $p(x)$. The MH algorithm uses a proposal distribution (pdf) denoted as $q(x)$. A candidate sample, x^*, drawn from the proposal distribution is either accepted or rejected depending on an acceptance ratio, A. In each iteration, the Markov chain moves to x^* if the sample is accepted. Otherwise, the chain remains at the current value of x. The algorithm proceeds for $N-1$ iterations to obtain N samples from the target distribution using the following steps.

1. Initialize the starting point x^0.
2. For $N-1$ iterations, complete the following four steps:
   a. draw a sample, x^*, from the proposal distribution; the pdf value is $q(x^*|x^i)$, where i denotes the current iteration and the distribution mean is x^i with a selected standard deviation
   b. sample u from a uniform distribution with a lower limit of zero and an upper limit of 1, $U(0, 1)$
   c. compute the acceptance ratio, $A = \min(1, (p(x^*)q(x^i|x^*)/p(x^i)q(x^*|x^i)))$, where $q(x^i|x^*)$ is the pdf value of the proposal distribution at x^i given a mean of x^* with the selected standard deviation, $p(x^*)$ is the pdf value of the target distribution at x^*, and $p(x^i)$ is the pdf value of the target distribution at x^i
   d. if $u < A$, then set the new value of x equal to the new sample, $x^{i+1} = x^*$; otherwise, the value of x remains unchanged, $x^{i+1} = x^i$.