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Re-examination of natural frequencies of marine risers by variational iteration method

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ABSTRACT

Marine risers are important components operating in offshore oil and gas industry. The vortex-induced vibration design of marine risers requires accurate knowledge of natural frequencies and mode shapes. Free vibration of marine risers are re-examined in this paper by means of variational iteration method, which is relatively new technique capable of dealing with eigenvalue problems rather efficiently. Solutions from the variational iteration method are compared to approximate solutions previously proposed in literatures via a numerical example. Furthermore, validation of the technique is demonstrated by comparing experimentally measured natural frequencies of model marine riser with the predicted ones.

1. Introduction

Marine risers are important component widely used in the offshore oil and gas industry for deep water exploration, production and transportation of petroleum products (Bai, 2001). Vibration due to vortex shedding leads to cyclic stresses and often develops into an unacceptable level of high cycle fatigue damage of risers. Avoiding vortex-induced vibration becomes an important consideration, which necessitate reasonable prediction of natural frequencies of marine risers (Iranpour et al., 2008).

Intensive research has been conducted on free vibration of marine riser in the last 30 years. Among the early studies, Dareing and Huang (1976) provided a power series solution for natural frequencies of riser with uniform flexural rigidity and mass density. Approximation assuming that the riser is subjected to uniform tension with one-half of the riser weight included was provided as well. In the meantime, the well-known Rayleigh–Ritz method was adopted by Kirk et al. (1979) to compute natural frequency of the riser, where the approximation was noted to be fairly accurate when compared to the power series solution. In a separated study, Kim and Triantafyllou (1984) attempted to use the Wentzel–Kramers–Brillouin (WKB) method to estimate the natural frequency in an implicit form. Another attempt was made by Soltanahmadi (1992) who used Fourier analysis to calculate natural frequencies of the riser. The approximation was noted to result in fairly accurate results. One other contribution on this subject is by Cheng et al. (2002) who proposed an approximate solution for natural frequencies by combing the dynamic stiffness method with the WKB theory. Their method was proved to be capable of yielding reasonably accurate results when compared with finite element solutions in previous literatures. Recently, an attempt was made by Chen et al. (2009) who introduced a semi-numerical–analytical method named differential transform method into the free vibration analysis of marine risers. Natural frequencies and mode shapes of marine risers were examined for various boundary conditions as well. To facilitate the natural frequency extraction, the marine riser may be treated as tensioned cable where the influence of flexural rigidity can be ignored due to their inherent slenderness (Chatjigeorgiou, 2008; Sparks, 2002; Graves and Dareing, 2004; Senjanovic et al., 2006). Therefore, approximate solution becomes obtainable by segmenting the cable and assuming that the tension is uniform and is equal to the average tension in each segment. Although this approach is considered efficient, the accuracy was found to degrade with higher mode frequencies.

A relative new technique named variational iteration method proposed by He (1999), He and Wu (2007), He (2007) has been proven to be capable of solving a large class of linear and nonlinear problems with approximations converging rapidly to exact solutions in a relatively easy way. This technique has been used with some success in solving various nonlinear problems arising in

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engineering. Abulwafa et al. (2006), Abulwafa et al. (2007) applied variational iteration method to nonlinear coagulation problem with mass loss and to nonlinear fluid flows in pipe-like domain, Ganji and Sadighi (2007) and Tari et al. (2007) to nonlinear heat transfer, Marinca (2002), Geng (2011) and Baghani et al. (2012) to nonlinear oscillators, Sweilam (2007) to nonlinear thermoelasticity, Liu (2005) to ion acoustic plasma wave, Siddiqui et al. (2006) to non-Newtonian flows, Batiha et al. (2007) and Wazwaz (2007) to linear and nonlinear wave equations, Berkani et al. (2012) and Maidi and Corriou (2013) to optimal control, Zhou and Yao (2010) to Cauchy problems, Wu and Baleanu (2013) to the Burgers' to flow with fractional derivatives, Liu and Gurram (2009) to free vibration of an Euler–Bernoulli beam, Coşkun and Atay (2009) to buckling load of columns and Pinarbasi (2011) to lateral torsional buckling of rectangular beams.

Despite the considerable advances made in characterizing the dynamic properties of marine risers, most of these methods are based on perturbation or discretion of the governing equation, often leading to tedious calculations. Furthermore, the efficiency and accuracy for those approximation methods for higher order eigenvalues sometimes becomes unpredictable. Whereas, variational iteration method provides an alternative solution with consistent efficiency and accuracy especially for higher order eigenvalues. While it is not the intent of this paper to expound all intricacies of the technique, the task at hand is to re-examine natural frequencies and mode shapes of marine riser via variational iteration method. Using a numerical example, approximations using variational iteration method are compared with solutions in available literatures. Furthermore, the accuracy of variational iteration technique is demonstrated by comparing experimentally measured natural frequencies with numerically estimated frequencies of model risers. The framework of application of variational iteration technique provided in this paper can be referred in engineering practice.

2. Basic idea of variational iteration method

As basic concepts in variational iteration are readily available in the literature by He (1999), only salient features are highlighted here for the application to free vibration analysis of marine riser. To illustrate the basic concept and idea of the variational iteration technique, we consider the following general differential equation:

$$Lu+Nu=g(x) \tag{1}$$

where L is a linear operator, N a nonlinear operator, and $g(x)$ an inhomogeneous term. According to variational iteration method, we can construct a correction functional as follows:

$$u_{n+1}(x)=u_n(x)+\int_0^x \lambda[Lu_n(s)+N\tilde{u}_n(s)-g(s)]ds \tag{2}$$

where λ is a general Lagrange multiplier, which can be optimally identified via the variational theory, the subscript n indicates the nth order approximation, $\tilde{u}_n(s)$ is considered as a restricted variation, i.e. $\delta u_n=0$.

Generally speaking, the use of the variational iteration method (VIM) follows the three steps: (a) to establish the correction functional; (b) identification of the Lagrange multipliers; (c) determination of the initial iteration. The success of the method mainly depends upon accurate identifications of the Lagrange multipliers. The principle of variational iteration method and its applicability for various kinds of linear and nonlinear differential equations are given in He and Wu (2007), He (2007). Some useful iteration formulae provided by He and Wu (2007) are listed in Table 1.

Table 1
Iteration formulae for the some typical differential equations (He and Wu, 2007).

No.	Iteration formulae
1	$\begin{cases} u'+f(u,u')=0 \\ u_{n+1}(t)=u_n(t)-\int_0^t \{u_n'(s)+f(u_n,u_n')\}ds \end{cases}$
2	$\begin{cases} u'+\alpha u+f(u,u')=0 \\ u_{n+1}(t)=u_n(t)-\int_0^t e^{\alpha(s-t)}\{u_n'(s)+\alpha u_n(s)+f(u_n,u_n')\}ds \end{cases}$
3	$\begin{cases} u'+f(u,u',u'')=0 \\ u_{n+1}(t)=u_n(t)+\int_0^t (s-t)\{u_n''(s)+f(u_n,u_n',u_n'')\}ds \end{cases}$
4	$\begin{cases} u''+\omega^2 u+f(u,u',u'')=0 \\ u_{n+1}(t)=u_n(t)+\frac{1}{\omega}\int_0^t \sin\ \omega(s-t)\{u_n''(s)+\omega^2 u_n(s)+f(u_n,u_n',u_n'')\}ds \end{cases}$
5	$\begin{cases} u''-\alpha^2 u+f(u,u',u'')=0 \\ u_{n+1}(t)=u_n(t)+\int_0^t \frac{1}{2\alpha}(e^{\alpha(s-t)}-e^{\alpha(t-s)})\{u_n''(s)-\alpha^2 u_n(s)+f(u_n,u_n',u_n'')\}ds \end{cases}$
6	$\begin{cases} u'''+f(u,u',u'',u''')=0 \\ u_{n+1}(t)=u_n(t)-\int_0^t \frac{1}{2}(s-t)^2\{u_n'''(s)+f(u_n,u_n',u_n'',u_n''')\}ds \end{cases}$
7	$\begin{cases} u^{(4)}+f(u,u',u'',u''',u^{(4)})=0 \\ u_{n+1}(t)=u_n(t)+\int_0^t \frac{1}{6}(s-t)^3\left\{u_n^{(4)}(s)+f(u_n,u_n',u_n'',u_n''',u_n^{(4)})\right\}ds \end{cases}$

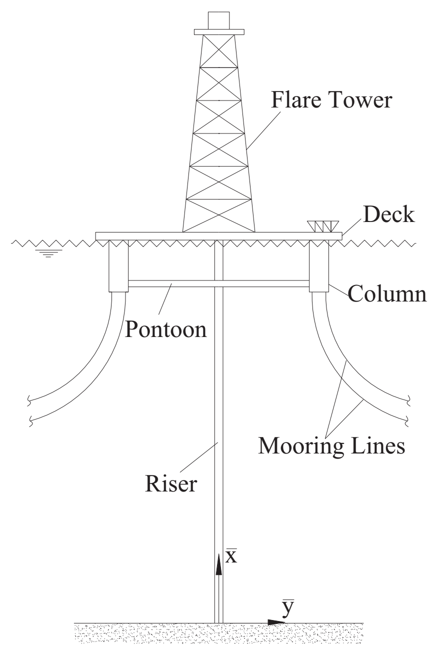

Fig. 1. An example of a marine riser connecting to a floating platform (adapted from Kyriakides and Corona (2007)).

3. Governing equation and boundary conditions

As commonly done in industry, marine riser usually can be simplified as a long, continuous tubular member that is straight and vertical in the whole length direction. The Cartesian coordinate system, defining the deformation of the riser, is shown schematically in Fig. 1, where the vertical coordinate $\bar{x}$ is measured from the bottom of the riser.

For a riser with varying flexural rigidity, axial force and mass density, governing equation for free vibration of the riser can be written in the form of following partial differential equation:

$$\frac{\partial^2}{\partial\bar{x}}\left(E(\bar{x})I(\bar{x})\frac{\partial^2\bar{y}}{\partial\bar{x}}\right)-\frac{\partial}{\partial\bar{x}}\left(T_e(\bar{x})\frac{\partial\bar{y}}{\partial\bar{x}}\right)+m_e(\bar{x})\frac{\partial^2\bar{y}}{\partial t^2}=0 \tag{3}$$

where $\bar{y}$ is the lateral deflection of the riser, which is a function of the vertical coordinate $\bar{x}$ and time t $E(\bar{x})I(\bar{x})$ is the flexural rigidity of the riser, $m_e(\bar{x})$is the effective mass (per unit length) participating in the vibration of the riser, $T_e(\bar{x})$ is effective axial force and can