Ocean Engineering 92 (2014) 75–82

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

A scaling law for form drag coefficients in incompressible turbulent flows

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ARTICLE INFO

Article history:
Received 7 June 2013
Accepted 28 September 2014
Available online 23 October 2014

Keywords:
Similarity laws
Eddy-making resistance
Form drag coefficient
Form factor
Scale effect

ABSTRACT

We present a similarity law for form drag coefficients, which is obtained by a judicious utilization of an energy dissipation equation due to eddy viscosity for incompressible turbulent flow, the steady-state κ–ω turbulence model, and the Kolmogorov turbulence dissipation length scales. It is shown that the form drag coefficients of three geometrically similar vessels subjected to turbulent flows are scaled according to $\overline{C}_p = \{c_1 + c_2\overline{Re}^{-1/3} + c_3\overline{Re}^{-1}\}$ where $(\overline{C}_p, \overline{Re}, c_i\ (i=1,2,3))$ are the form drag coefficient ratio, the mean flow Reynolds number ratio, and c_i are closure coefficients to be determined from existing geometrically similar vessels, respectively. The present theoretical form factor methodology is applied to predict the full scale form factor from the scaled towing tank experiment based on three simulation results, which show improved correlations compared to the empirical least-squares prediction methods.

1. Introduction

A major goal in the design of aerospace and/or marine vehicles is to reduce drag emanating from skin friction as well as form drag (or pressure drag). Thanks to the century-old legacy of aerospace and marine vehicle design activities, there exist a host of base-line designs from which one can improve the efficiency of existing geometrically similar vehicle types, or design a new class of *scaled-up* vehicles. The concurrent increase in both the vehicle size and the operation speed inherently subject the modern air and marine transportation vehicles, as well as wind energy blades, to turbulent flow ranges. In scaling-up design iterations, an important design parameter that is needed is drag coefficients due to skin friction and form drag. Skin friction (more precisely skin friction coefficient) for a new design can be estimated based on the power and/or logarithmic laws (Hinze, 1975; ITTC, 1957; Prandtl, 1905; Schlichting, 1979; Prandtl, 1921; von Karman, 1934), among others. Hence, designers can estimate the drag due to skin friction in their assessment of overall aerodynamic or hydrodynamic performance.

When the aspect ratio (i.e., thickness-to-length ratio) is small such as thin plates, the form drag remains insignificant. However, when the vehicle cross section bulges out significantly as typically the case for surface ships, the form drag constitutes a significant part of the total drag loss. It is generally accepted that the form drag in turbulent flows is caused by the viscous pressure applied on a vehicle surface. However, there exist scant proposed formulas or rational estimation procedures for the estimation of form drag coefficient. In ship hydrodynamics, for example, the prevailing ITTC practice determines the total viscous drag coefficient from a scaled-model test in which the wave-making resistance can be considered minimal or non-existent. The difference between the ITTC1957 skin friction line and the viscous drag coefficient is then deemed to be due to the form drag. This form drag coefficient which is expressed as a percentage of the skin friction coefficient, known as *form factor*, is then assumed to remain constant for the full model (ITTC, 1957). In an effort to improve the prediction of form factors, several investigators proposed various schemes ranging from a least-squares data fit (García-Gómez, 2000; Min and Kang, 2010) to CFD-based prediction (Kim and Menon, 1997; Kouh et al., 2009). To date, there exist no comparable theory or formula that one can rely to estimate form factors as is the case for skin friction.

The present study is an attempt to develop an engineering scaling law for estimating the power loss due to form drag in turbulent flows, which may be utilized in the design of geometrically similar full model vehicles, provided there exist form drag data for geometrically similar scale-model vehicles. In so doing, we observe that energy loss consists of two sources: the first due to skin friction near the wall/boundaries and the second due to the energy dissipation of locally isotropic Kolmogorov-scale (η) turbulence away from the boundaries or simply due to eddy viscosity.

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http://dx.doi.org/10.1016/j.oceaneng.2014.09.044
0029-8018/

As for the skin friction loss, we accept that the near wall/boundary energy loss is accounted for by relying on the ITTC skin friction formula for surface vessel design. As for energy loss due to the viscous pressure – caused form drag in turbulent flows, we stipulate that the bulk of energy loss due to viscous pressure – caused form drag – is associated with eddy viscosity dissipation away from the submerged vessel surface. In other words, the dominant energy loss mechanisms away from the wetted surface are due to eddy viscosity dissipation, which in turn accounts for energy loss due to viscous pressure – caused form drag. This stipulation allows us to relate the form drag to the energy loss terms in turbulence in the energy equation (see, e.g., Stewart, 1942; Stewart and Townsend, 1951; Daly and Harlow, 1970; Wilcox, 2008).

It turns out that, in order to relate the energy loss terms of one scale to those of another scale, one needs the scaling relations of the kinematic eddy viscosity (ν_t), the turbulence kinetic energy (k), the Kolmogorov eddy scale (η), the mean flow length scale (ℓ_u), the mean flow velocity (U), and their interrelations. This puts us to employ a suitable turbulence model. We note that, although the turbulence kinetic energy (TKE) equation should, in principle, provide the needed interrelations among the preceding five variables, the TKE closure problem remains open problems; hence, we are unable to employ it in the present study. Among several proposed turbulence models (see, e.g., Landau and Lifshitz, 1959; Smagorinsky, 1963; Launder and Spalding, 1974; Hinze, 1975; Launder et al., 1975; Launder, 1989; Lilly, 1992; Spalart and Allmaras, 1992; Menter, 1992, 1994), we have chosen the k–ω turbulence model (Kolmogorov, 1942; Saffman, 1970; Wilcox, 1988, 2004), primarily because of our familiarity with it. We are aware of that what we are adopting is an isotropic turbulence model, not anisotropy turbulence theories (Biferale and Procaccia, 2005; Tong et al., 1990), by invoking the Kolmogorov hypothesis, viz., in turbulent flows, energy dissipation occurs in the Kolmogorov eddy scales for which isotropic turbulence assumption is assumed to be valid. Should anisotropic/intermittency theories prove to be applicable for form drag estimation, we believe the present methodology would apply. The rest of the paper is organized as follows.

Section 2 begins with the resistance expression that consists of the skin friction resistance and form drag (or viscous pressure) resistance. The ratio of the form drag for two geometrically similar vessels are then related to the ratio of the energy loss expressions. It is this ratio that the present paper seeks to determine.

Section 3 introduces the k–ω turbulence model together with the Kolmogorov scale to extract similarity-obeying conditions among the five variables ($\nu_t, \eta, k, \ell_u, U$). It is found that the k–ω turbulence model does not satisfy complete similarity requirements. To alleviate this difficulty, it is rearranged in powers of the small parameter $\epsilon_t = Re^{-1} \ll 1$ where Re is a characteristic Reynolds number. Similarity laws are then applied separately to each power of ϵ_t, which is analogous to a procedure adopted in classical asymptotic nonlinear analysis. A surprisingly simple relation is obtained for the ratio of eddy to mean-flow length scales given by $\overline{\ell}_t/\overline{\ell}_u = \overline{Re}^{-1/3}$ where the over bar denotes the ratios of appropriate variables for two geometrically similar vessels.

Section 4 derives a theoretical ratio of the form drag coefficients of two geometrically similar vessels solely in terms of their Reynolds number ratio. Section 5 summarizes the main results of the present study along with limitations of the present results and future work being carried out.

2. Problem statement

Our objective is to find a scaling law for the total resistance (R_{total}) consisting of the wave-making resistance (R_W), the resistance due to skin friction (R_f) and due to form (or viscous pressure) drag emanating from the energy dissipation associated with the eddies (R_p) in incompressible turbulent flows. To this end, we express the total resistance (R_{total}) as

$$R_{total} = R_W + R_V$$
$$R_V = R_f + R_p = C_V \tfrac{1}{2}\rho U^2 A, \quad C_V = C_f + C_p \tag{2.1}$$

where $(C_W, C_V, C_f, C_p, \rho, U, A)$ denote the wave-making resistance coefficient, the viscous resistance coefficient, the skin friction resistance coefficient, the form drag (or viscous pressure resistance) coefficient, the density, the mean-flow velocity, and the wetted surface area of the vessel of interest, respectively.

For subsequent analysis, we focus on the viscous drag, assuming that the viscous drag can be obtained either by assuming that the wave-making resistance is known or it can be effectively neglected by a careful low speed setup of the towing tank experiment. Hence, from now on we limit ourselves to the viscous drag only.

For a class of geometrically similar vessels, we wish to find a scaling functional ($\overline{C}(\overline{Re}, \overline{\ell})$) that relates the viscous resistance coefficient of one scale to that of another scale in the form of

$$C_V^{(a)} = \overline{C}(\overline{Re}, \overline{\ell}) C_V^{(b)}, \quad \overline{Re} = \frac{Re^a}{Re^b}, \quad \overline{\ell} = \frac{\ell^a}{\ell^b} \tag{2.2}$$

and similarly for the skin friction and form drag coefficient, where superscripts (a, b) refer to two different geometrically similar vessels, and (ℓ^a, ℓ^b) are characteristic lengths of the two vessels.

In this paper we adopt the log-law skin friction coefficient (C_f) that is expressed as (cf., Schlichting, 1979)

$$C_f = \alpha(\log_{10} Re - \beta)^\gamma \quad (R_f = C_f \tfrac{1}{2}\rho U^2 A) \tag{2.3}$$

where (α, β, γ) are constants proposed by various investigators.

What remains to be done is the determination of the form drag coefficient (C_p) that is needed for the computation of the form drag (or eddy-making resistance) expressed as

$$R_p = C_p \tfrac{1}{2}\rho U^2 A \tag{2.4}$$

The difficulty in expressing the eddy-making resistance in terms of mean-flow speed (U) is that the turbulence kinetic energy (TKE) associated with the eddies (k) is given by

$$k = \frac{1}{2}\sum_{i=1}^{3} \langle u_i' u_i' \rangle \tag{2.5}$$

where u_i' is the fluctuating turbulence velocity, and $\langle \cdot \rangle$ denotes averaging operator.

Since our objective is to find a scaling law governing the eddy-making resistance coefficients, we express the ratio of form drag coefficients from (2.4) as

$$\overline{C}_p = \frac{C_p^s}{C_p^f} = \left[\frac{R_p^s}{R_p^f}\right]\left[\frac{1}{\overline{\rho}\,\overline{U}^2 \overline{A}}\right], \quad \overline{\rho} = \frac{\rho^s}{\rho^f}, \quad \overline{U} = \frac{U^s}{U^f}, \quad \overline{A} = \frac{A^s}{A^f} \tag{2.6}$$

where superscripts (s, f) refer to two geometrically similar scales.

The above equation, circuitous it may seem, leads us to find $\overline{C}_p$ if we know the ratio of the eddy-making resistance. This is derived as follows.

The energy conservation for incompressible flows can be expressed as (Stewart, 1942; Wilcox, 2008)

$$\frac{\partial}{\partial t}\left[\rho\left(\frac{1}{2}u_i u_i + k\right)\right] + \frac{\partial}{\partial x_j}\left[\rho u_j\left(\frac{1}{2}u_i u_i + k\right)\right] = \frac{\partial}{\partial x_j}\left[u_i(\tau_{ij}^u + \rho\tau_{ij}^t) + \rho(\nu + \sigma^*\nu_t)\frac{\partial k}{\partial x_j}\right] \tag{2.7a}$$

$$\tau_{ij}^u = 2\mu S_{ij}, \quad \tau_{ij}^t = 2\nu_t S_{ij} - \frac{2}{3}k\delta_{ij}, \quad S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{2.7b}$$