Ocean Engineering 91 (2014) 51–63

Contents lists available at ScienceDirect

Ocean Engineering

journal homepage: www.elsevier.com/locate/oceaneng

# A multi-fluid modelling approach for the air entrainment and internal bubbly flow region in hydraulic jumps

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ARTICLE INFO

*Article history:*
Received 4 March 2014
Accepted 28 August 2014

*Keywords:*
Hydraulic jump
Air entrainment
Bubbly flow
MUSIG

ABSTRACT

This paper proposed a numerical modelling framework aiming to couple both Volume-Of-Fluid (VOF) model and MUltiple-SIze-Group (MUSIG) model to handle the formation of large-scale free surface, bubble entrainment and bubble dispersion in hydraulic oscillating jumps. To consider all essential physics, the modelling framework resolves and couples the flow structure of three different fluids (i.e. continuous air and water and dispersed air bubbles) using the Eulerian–Eulerian multi-fluid approach. To model the air entrainment at the jump toe, a sub-grid scale air entrainment model was also implemented within the framework. To evaluate the capability of the proposed model, model predictions were validated against experimental data of (Chachereau and Chanson, 2010). Comparisons between predicted and measured results are in satisfactory agreement demonstrating the potential of the proposed methodology. Discussions on the drawbacks and deficiencies of the current model are also included.

## 1. Introduction

Hydraulic jump is a natural flow characteristic which is frequently encountered in many hydraulic structures, industrial open channels and manufacturing processes. In general, a hydraulic jump involves complex flow energy dissipation and transition from supercritical to subcritical flow region; resulting in a sudden velocity reduction and increment in depth in flow direction. Although hydraulic jump could occur in very low Reynolds number laminar flow, most practical applications involve turbulent flows where more efficient energy dissipation is favourable to limit the erosion of structures and minimize the probability of damage due to cavitation. Due to its wide applications, turbulent hydraulic jump has been studied for centuries; where the earliest study can be dated back to Leonardo Da Vinci in the sixteenth century (Rouse et al., 1957). Based on previous studies, it is well-known that the hydraulic jump can be classified in according to its inflow Froude number above unity. For Froude number slightly above unity, it is characterized by a smooth rise followed by a series of stationary free-surface undulations (Chanson, 2009). Experimental measurements and observations showed that the free-surface undulations are quasi-periodic with smooth and clear interface between air and water. It is also why some literatures refer it as–non-breaking undular hydraulic jump. For higher Froude numbers (i.e. Fr=2.5–4.5), the hydraulic jump becomes unsteady with significant kinetic energy dissipation and a bubbly two-phase flow region (also refers as “Oscillating jumps”). A schematic diagram of the flow structure in a typical oscillating jump is shown in Fig. 1. As depicted, the flow structure of an oscillating jump can be broadly divided into three regions. In the supercritical region, the inflow velocity is gradually reduced due to the energy dissipation and boundary layer development under the free-surface flow. At the onset of hydraulic jump (i.e. jump toe), rigorous kinetic energy dissipation occurs in the form of turbulent rollers. Meanwhile, significant amount of air bubbles and air pockets are also entrained at the jump toe due to the impingement of inflow velocity (Chanson, 2010). The entrapped air bubbles are then advected within the shear layer region forming a bubbly two-phase flow downstream in the subcritical flow region. Within the subcritical flow region, migration of air bubbles and the spatial distributions of air void fraction are then coupled with the internal flow structure through buoyancy and interfacial forces. Furthermore, local bubble number density and bubble sizes are also governed by bubble coalescence and breakage process which eventually have strong implications in the air–water mixing and mass transfer processes. Theoretically speaking, the aforementioned flow structures are extremely complex and closely coupled

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http://dx.doi.org/10.1016/j.oceaneng.2014.08.016
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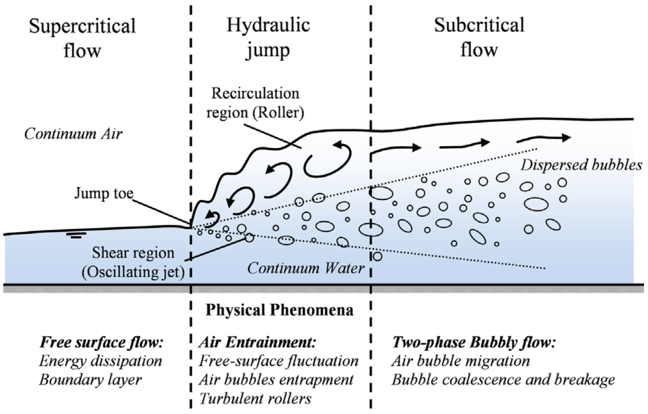

**Fig. 1.** Schematic diagram of flow structure in typical hydraulic jumps.

physical phenomena occurring in multiple forms of fluids (e.g. continuum air, continuum water and dispersed air bubbles). The underlying physics of these phenomena remain largely elusive.

Although most of the earlier studies were focused only on the external water flow structure (Leutheusser and Kartha, 1972; Rajaratnam and Subramanya, 1968), the first detailed internal velocity measurements of hydraulic jumps were obtained by Rouse et al. (1957) using hot-wire anemometry. Rajaratnam (1965) then carried out the internal measurement using a Pitot tube, concluding that the velocity distributions were similar to that in a typical wall jet. Babb and Aus (1981) developed gamma ray and hot-film anemometer methods for determining void fraction and sizes of air bubbles entrained in hydraulic jump flow. Recently, with the advancement of experimental measuring technique, several comprehensive experimental works have been carried to investigate the internal flow structure; including characteristics of the velocity field, turbulence vortices and air entrainment processes.

Chanson and Brattberg (2000) conducted an experimental to measure air–water flow properties using dual-tip conductivity probes to investigate the bubbly flow region; including vertical void fraction, bubble frequency profiles and velocity distributions. They concluded that the hydraulic jump is characterised by two air–water flow region with significantly different properties. Murzyn et al. (2005) used an optical probe to investigate the bubble characteristics, their findings revealed that the void fraction profiles exhibit distinguished characteristics between upper and lower part of the jump. They also demonstrated the significance of bubbly flow region and its associated turbulence-bubble interactions on the overall flow structure. Afterwards, Chanson (2010) carried out a large-scale experiment studying hydraulic jumps at large Froude numbers. Based on the measurements, he concluded that the turbulent air bubble mixing coefficient has a linear relationship with the distance from jump toe.

Recently, Lin et al. (2012) measured the internal flow structure in a steady hydraulic jump using the image-based particle and bubble image velocimetry techniques and successfully obtained the ensemble averaged mean velocities and turbulence statistics from a series of repeated measurements. It is no denying that the above experimental works have provided valuable insight on the bubbly flow region and its influence on the internal flow structure. Nevertheless, owing to the limitation of current measuring techniques, experimental measurements are still very limited. The reported findings and observations remain inconclusive and inconsistent. More research efforts are due to be made to obtain more accurate and detailed data.

On the other hand, with the rapid advancement of computational technology, numerical modelling has fast become an effective tool to complement the deficiencies of existing experimental techniques. Nevertheless, the complexity of air entrainment and bubbly flow region in hydraulic flows pose a great challenge for the model development. Several researchers (Ma and Hou, 2001; Passandideh-Fard et al., 2007; Sabbagh-Yazdi SR, 2007) attempted to capture the free-surface external flow characteristic using various interface capturing techniques (e.g. Volume of fluid and Level set approaches). The free-surface profile and the jump formation characteristics of different types of hydraulic jump were successfully captured. On the other hand, instead of free surface tracking technique, Cheng and Chen (2011) proposed an improved drag model to characterize the interfacial momentum transfer for the free surface and dispersed gas bubbles using Eulerian–Eulerian framework. Although the predicted air void fraction distributions and velocity profiles were in reasonable agreement with experimental data, air entrainment and its influence on the bubbly flow region were ignored in simulations.

In order to model the air entrainment rate, several numerical models have been proposed to consider various air entrainment mechanisms. Based on the observation from their experiments, Ma et al. (2010) proposed an empirical air entrainment model to study the process of air ingestion due to plunging jet. Similarly, empirical approach was also adopted by Xiang et al. (2011) to investigate the gas pocket leakage process due to re-entrained jet. Skartlien et al. (2012) developed a semi-empirical model to investigate the three essential mechanisms (i.e. plunging entrainment, gas entrapment and gas leakage) of air entrainment in the hydraulic jump. Satisfactory results were obtained for a range of inflow velocity and pipe diameter. Unfortunately, their research work only focused on quantifying the entrained gas flux with respect to inflow velocity. Detailed internal flow structure in hydraulic jump was not considered. Lately, Waltz (2008) proposed a more comprehensive numerical scheme aiming to incorporate the air entrainment mechanism into a generic two-fluid model in conjunction with Volume Of Fluid (VOF) surface capturing technique. The predicted void fraction, velocity and turbulence dissipation rate distributions were compared reasonably well with experimental data. Ma et al. (2011) adopted similar numerical scheme but using level set free surface model instead to predict the overall void fraction distributions in hydraulic jumps. However, the air bubble size evolution due to coalescence and breakage processes were not considered in both studies.

This paper presents a numerical framework attempting to incorporate all essential physical considerations to investigate the internal flow structure of hydraulic oscillating jumps. The numerical framework is developed based on the Eulerian–Eulerian multi-fluid model; which solves the phasic distribution of fluids explicitly through fundamental interfacial momentum transfer models. Air ingestion at the jump toe is handled by a sub-grid air entrainment model while the location of free surface is captured using the compressive VOF model. More importantly, for the very first time, the MUltiple-SIze-Group (MUSIG) model (Cheung et al., 2007; Xiang et al., 2011) together with mechanistic coalescence and breakage kernels are also included in the calculation to better represent the air bubble size evolution in the subcritical flow region. The predicted is then validated against the recent experimental data reported by Chachereau and Chanson (2010). Discussions on the numerical results are presented in later sections.

## 2. Mathematical models

### 2.1. Multi-fluid model framework

The formation of hydraulic jump flow can be considered as the interaction of three different phases: continuous liquid, continuous gas above the free-surface and disperse air bubbles. In this paper, numerical model is established based on the Eulerian–Eulerian