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Identification of passive state-space models of strongly frequency dependent wave radiation forces

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ABSTRACT

This paper describes a methodology for obtaining passive state space representations of wave radiation forces for floating bodies with zero forward speed oscillating in multiple degrees of freedom. The method is based on fitting rational transfer functions written on pole-residue form to radiation frequency responses calculated using standard boundary element codes. Determination of poles and residues is facilitated by the vector fitting algorithm, originally developed for fitting of frequency responses of electrical networks. Using the poles and residues as parameters in the fitting has advantages over the more common ratio-of-polynomials transfer function formulation when the model order is high, which is typically required when the frequency dependence is strong and wide-banded such as for multibody floating systems. A methodology for perturbing the residues of slightly non-passive systems such that they become passive is also presented. The method is demonstrated successfully for a five-body wave energy converter, an array of circular cylinders and a single cylinder. A discussion and investigation of the truncation of high frequencies is provided for the wave energy converter, and parameter constraints governing the extrapolation of the rational model above the frequencies present in the dataset are suggested.

1. Introduction

Linear potential flow theory is still an important tool in the analysis of wave–body interactions. For linear equations of motion, a frequency domain analysis is the preferred choice since statistical response quantities are then readily obtained from the frequency response functions and the wave spectrum. If the equations of motion include nonlinear forces in addition to the linear hydrodynamic ones, a time-domain approach is needed. In the latter case, the motions are given by Cummins equation (Cummins, 1962), which includes a convolution integral representing the fluid memory effect. The kernel of the convolution integral is the retardation function, which can either be obtained directly from time domain potential flow solvers (Babarit, 2010) or (more often) from its frequency domain counterparts, the added mass and damping, using the Ogilvie relations (Ogilvie, 1964).

Cummins equation is inconvenient to implement in standard simulation packages and the convolution integral is time consuming to evaluate. Moreover, it is not convenient for analysis and design of control systems (Taghipour et al., 2008). For these reasons, many authors have sought a replacement of the convolution integral by an approximate state space model, with the advantage that the equations of motions are reduced to ordinary differential equations. The task of finding the approximate state space model based on another type of system description can be seen as a system identification problem. Taghipour et al. (2008) reviewed and compared different system identification approaches in a hydrodynamic context. The three approaches considered were impulse response curve fitting, realization theory and regression in the frequency domain. He demonstrated that all these methods work well for identification of a radiation force model of a modern container-ship with zero forward speed in open sea conditions, and that they give large savings in computational time as compared to the direct application of Cummins equation. A single body in open sea conditions without moonpools or other confined water volumes inside is characterized by that the added mass and damping have only a few peaks along the frequency axis and that the retardation function has a relatively short memory. Multibody systems, single bodies in confined water volumes (e.g. towing tanks) and other cases where resonant fluid motion occur are on the contrary characterized by added mass and damping curves with a large number of peaks along the frequency axis and a retardation function with a long memory.

This paper will demonstrate a frequency domain identification method based on fitting of rational transfer functions written on a

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pole-residue form, which is particularly well suited for problems with strongly frequency dependent added mass and damping. The method will be demonstrated for a five-body wave energy converter system and an array of upright circular cylinders with equal spacing, closely related to the channel problem. The method has been developed through a series of papers (Gustavsen and Semlyen, 1999; Gustavsen, 2006; Deschrijver et al., 2008) and implemented in a package of Matlab routines made available on the Internet (Gustavsen, 2012). The method was originally intended for analysis of electrical power systems, and to the authors knowledge, it has not been reported in the hydrodynamic literature.

For large multibody systems, the evaluation of added mass and damping for high frequencies can easily become infeasible since the panel size used in the boundary element solvers should be taken proportional to the wave length associated with the highest frequency. For this reason, the effect of the high-frequency truncation is investigated and a new parameter constraint governing the behavior of the high-frequency tail, above the fitting range, is suggested.

Passivity is a fundamental property of wave radiation forces which we want to retain in the approximating state space model. This property implies that the *time averaged* energy transport from the body (or system of bodies) is non-negative. For the high order models needed to represent strongly frequency dependent radiation forces, small passivity violations are hard to avoid. An accompanying method for passivity enforcement is developed in the papers (Gustavsen, 2008; Semlyen and Gustavsen, 2009). In this paper, the method for passivity enforcement has been modified and re-implemented to suit systems where the added mass and damping matrices are sparse and structured, which is typical for multibody systems with geometrical symmetry. With the modified method, this sparsity and structure is retained after the passivity enforcement. A modification to the passivity assessment method, which is necessary for systems with vanishing damping in the low-frequency limit, is also suggested.

2. Important properties of wave radiation forces

Cummins equation (Cummins, 1962) governs the motions of a floating body at zero forward speed in the time domain and can be written as

$$(\mathbf{M}+\mathbf{A}_{\infty})\ddot{\mathbf{q}}(t)+\int_0^t \mathbf{k}(t-\tau)\dot{\mathbf{q}}(\tau)\,d\tau+\mathbf{C}\mathbf{q}(t)=\mathbf{f}_e(t)+\mathbf{f}_{nl}(t,\mathbf{q},\dot{\mathbf{q}}) \tag{1}$$

where we have included an additional term, $\mathbf{f}_{nl}(t,\mathbf{q},\dot{\mathbf{q}})$, representing external, generally non-linear forces. $\mathbf{f}_e(t)$ are the linear wave excitation forces and $\mathbf{A}_{\infty}$ is the added mass matrix in the limit where the frequency tends to infinity. Instead of representing $\mathbf{A}_{\infty}$ by a high-frequency approximation, which would require a very fine discretization of the body geometry, one should find $\mathbf{A}_{\infty}$ by solving a modified boundary value problem where a condition of vanishing velocity potential on the free surface is included. The commercial BEM code WAMIT, which is used in this study, has this ability.

The retardation function $\mathbf{k}(t)$ can be obtained directly from time domain potential flow solvers (see e.g. Babarit, 2010), or it can be found from frequency domain data using the inverse Fourier transform

$$\mathbf{k}(t)=\mathcal{F}^{-1}[\mathbf{K}(j\omega)]=\mathcal{F}^{-1}[\mathbf{B}(\omega)+j\omega(\mathbf{A}(\omega)-\mathbf{A}_{\infty})] \tag{2}$$

Here, $\mathbf{B}(\omega)$ and $\mathbf{A}(\omega)$ are the damping and added mass, respectively. The retardation function $\mathbf{k}(t)$ is causal, implying that $\mathbf{k}(t)=\mathbf{0}$ for $t<0$ (i.e. there is no force due to future motions). A consequence of this is that we can find $\mathbf{k}(t)$ with knowledge only about the damping

$$\mathbf{k}(t)=\frac{2}{\pi}\int_0^{\infty}\mathbf{B}(\omega)\cos(\omega t)\,d\omega, \tag{3}$$

or only the added mass

$$\mathbf{k}(t)=-\frac{2}{\pi}\int_0^{\infty}\omega(\mathbf{A}(\omega)-\mathbf{A}_{\infty})\sin(\omega t)\,d\omega \tag{4}$$

These are often referred to as the Ogilvie relations (Ogilvie, 1964). For further reading, Falnes (2002) gives a thorough discussion about impulse response functions in hydrodynamics and their causality properties.

The asymptotic values of the retardation function $\mathbf{k}(t)$ as $t\to 0$ and $t\to\infty$, and its Fourier transform $\mathbf{K}(j\omega)$ as $\omega\to 0$ and $\omega\to\infty$ are given by e.g. Taghipour et al. (2008)

$$\mathbf{k}(0)=\frac{2}{\pi}\int_0^{\infty}\mathbf{B}(\omega)\,d\omega\neq\mathbf{0}<\infty \tag{5}$$

$$\lim_{t\to\infty}\mathbf{k}(t)=\mathbf{0} \tag{6}$$

$$\mathbf{K}(0)=\mathbf{0} \tag{7}$$

$$\lim_{\omega\to\infty}\mathbf{K}(j\omega)=\mathbf{0} \tag{8}$$

It is worth noting that although $\mathbf{K}(0)=\mathbf{0}$, we have that $\mathbf{A}(0)\neq\mathbf{0}$. Further, both the real and the imaginary part of $\mathbf{K}(j\omega)$ is symmetric

$$K_{ij}(j\omega)=K_{ji}(j\omega) \tag{9}$$

The proof of this symmetry property can be found in e.g. Falnes (2002).

The wave radiation forces are passive. This is a fundamental property which implies that the *time averaged* energy transport from the body is non-negative. The mathematical condition for passive wave radiation forces is that the damping matrix is positive semi-definite, meaning that

$$\mathbf{x}^T\mathbf{B}\mathbf{x}\geq 0 \tag{10}$$

holds for any $\mathbf{x}$ (Falnes, 2002, p. 213). A necessary, but not sufficient, condition for positive semi-definiteness is non-negative diagonal entries. When $\mathbf{B}=\mathrm{Re}[\mathbf{K}]$ is positive semi-definite, we say that $\mathbf{K}$ is positive real.

If we apply a power conservative transformation (see e.g. Karnopp et al., 2006) $\mathbf{B}'=\mathbf{T}^T\mathbf{B}\mathbf{T}$ where $\mathbf{T}$ is a square matrix with linearly independent columns, positive definiteness of $\mathbf{B}'$ implies positive definiteness of $\mathbf{B}$. Moreover, if we choose $\mathbf{T}$ to be the matrix whose columns are the eigenvectors of $\mathbf{B}$, we know that $\mathbf{T}^T=\mathbf{T}^{-1}$, because $\mathbf{B}=\mathbf{B}^T$, and that $\mathbf{B}'$ becomes diagonal with B'_{ii} equal to eigenvalue i of $\mathbf{B}$. Thus, we can formulate an equivalent passivity criterion which is easier to implement in calculations

$$\mathrm{eig}(\mathbf{B})\geq\mathbf{0} \tag{11}$$

Here, $\mathrm{eig}(\mathbf{B})$ is a vector containing the eigenvalues of $\mathbf{B}$.

Khalil (2002) defines a passive system to be one that satisfies $\mathbf{f}(t)^T\dot{\mathbf{q}}(t)\geq\dot{V}(t)$, where $\dot{\mathbf{q}}(t)$ is the system inputs (here velocities), $\mathbf{f}(t)$ is the system outputs (here forces) and $\dot{V}(t)$ is the rate of change of some scalar storage function (here stored energy). The added mass is associated with kinetic and potential energy stored in the body's near field, whereas the damping is associated with energy radiated into the far-field (Falnes, 2002). Thus, if we define $V(t)$ to be the kinetic and potential energy in the near field, the condition of positive semi-definite damping matrix complies with Khalil's definition of passivity.

3. Utilizing geometrical symmetries

Floating bodies with geometrical symmetries will have added mass and damping matrices with a special structure. One example