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Design, manufacturing, and performance verification of a Roberts linkage for inertial isolation

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ABSTRACT

There are a number of experimental apparati aimed at the measurement of small forces, whose performance is strictly related to the availability of an isolated inertial frame. This means that one component of the apparatus has to be set in free-fall condition along selected degrees of freedom. Such a free-fall condition can be achieved thanks to dedicated mechanisms acting as inertial isolation systems. The present work discusses the optimal design, realization, and testing of an inertial isolation system implemented as a 3-D Roberts linkage. The first part of the work describes the design phase: the optimal calculation of the linkage design parameters is performed through the application of a Nelder–Mead optimization scheme. In order to apply the optimization scheme, the calculation of the linkage kinematics is firstly performed through a variational approach, then the sampled surface describing the linkage trajectory is approximated by a set of Zernike polynomials, which allows to effectively define and calculate the target function for the optimization itself. The second part of the work describes the detailed design and realization of a linkage prototype, and its chararacterization by means of a coordinate measuring machine. The characterization of the device is carried out by exploiting a coordinate measuring machine, which is unusually employed for both driving the linkage moveable part along its two DoF trajectory, and for measuring the resulting error about the nominal trajectory.

1. Introduction

In many different fields the need arises to minimize undesired forces acting on a mechanical system, typical examples of stray forces being seismic motion (often the most pervasive and difficult to limit), air flow and turbulence, pressure waves, and local electro-magnetic fields. For example, a vibration isolation system [1] makes it possible to limit the effect of seismic noise on a building, in order to prevent it from high inertial loads and subsequent structural damage. Similarly, more refined isolation systems are adopted to suspend from ground vibration very sensitive instruments, like atomic force microscopes, nanoindenters, surface force apparati, etc. In these cases, the strategy for isolation from vibration is to insert a damped mechanical filter between the system and ground, such that the seismic random motion is only partially transmitted. As a result, the suspended system is kept still enough that the effect of inertia forces becomes negligible.

Other applications require isolation systems of higher complexity. For example, small forces or impulses can be detected through very precise measurement of the motion of suspended bodies that must show a mechanical impedance—as seen by the force source—as close as possible to the free-falling one. This additional requirement drives the design of a dedicated, *undamped* mechanical filter (inertial isolation filter) that usually follows a seismic isolation system [2]. This philosophy is largely adopted in the design of ground-based gravitational waves detectors, where the sensitive elements are set to a free-falling condition along the directions orthogonal to the gravity force, such that—above a certain frequency—their motion becomes sensible to the suspension thermal noise [3].

The achievement of the free-falling status on ground is usually pursued for one degree of freedom, along which the force or impulse to be measured is applied. In this case, the preferred option is to adopt torsion pendula, which take advantage of both the low torsional stiffness of thin wires and the relatively low rotational micro-seismic activity of the ground about a vertical axis [4,5]. The resulting free-falling degree of freedom of the suspended equipment is the rotation around a vertical axis, that may be converted into horizontal displacement by means of a proper lever arm.

It is worth noting that, even in apparati where the suspended system only needs one single degree of freedom set in free-fall, in order to limit cross-talk noise from the other degrees of freedom the seismic isolation must filter the ground noise coming from all the six degrees of freedom. The capability to decouple a load with respect to ground is therefore instrumental both to the

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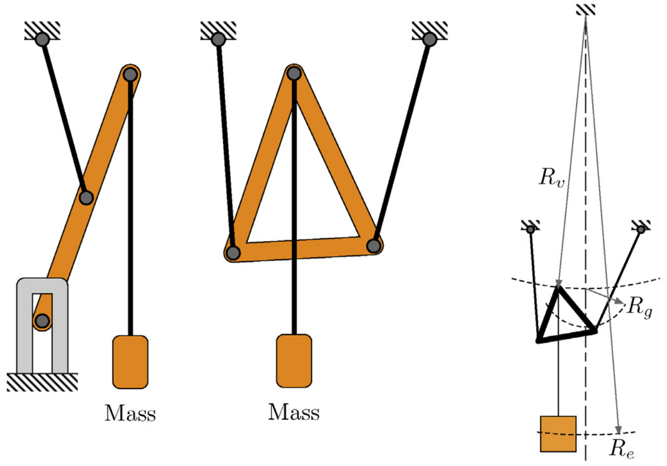

Fig. 1. Schemes for Scott-Russel (left) and Roberts (middle) linkages, and centers of instantaneous rotation (right).

isolation from seismic vibration and to the realization of a free-fall status.

Consequently, since it is commonly recognized that the disturbance from horizontal seismic motion represents the main limit to the isolation that can be obtained at low frequencies [6], it is particularly important to uncouple the horizontal displacements by suspending the system with a simple pendulum stage or a mechanism that realizes a large-radius curved motion along the two directions orthogonal to the gravity force. When the mass to be isolated moves on the generated trajectory by the mechanism, it finds itself to swing in a very shallow gravitational potential well, resulting weakly coupled to ground through a low-frequency oscillator. Needless to say, the larger the pendulum radius, the shallower the potential well, and the softer the constraint.

Furthermore, in order to limit the size of the apparatus, linkage mechanisms have been conceived so to increase as much as possible the ratio between the radius of curvature of the potential well over the typical dimension of the apparatus implementing it. This requirement is particularly relevant in the field of ground-based gravitational wave detectors, where the highest noise rejection performances must be realized by minimizing weight and dimensions, in order to fit with the available workspace of vacuum chambers. We assume hereinafter these applications as reference cases for the proposed design and verification method, not excluding that similar considerations may apply to other fields where high-performance compact inertial isolation systems are needed.

So far, many mechanisms have been exploited to generate nearly planar trajectories. A Chebyshev linkage has been developed in order to provide a low-frequency oscillator along one axis [7]. Another solution employed a four-bar linkage to provide a low-frequency oscillator and could reach a 20 mHz resonant frequency [8]. Both solution are limited by the fact that they provide isolation along a single degree of freedom, whereas many applications require at least two isolated axes.

The Scott-Russel linkage shown in Fig. 1 (left) has been adopted with very good isolation performance in terms of horizontal noise rejection ratio. A prototype of the mechanism has been operated with a resonant frequency down to 50 mHz with an amplitude of the motion between 10 and 30 mm [6]. In the isolation stage developed by Winterflood et al. [9] the resonant frequency has been pushed down to less than 10 mHz, with particular care for ground tilt noise feedthrough. It has to be noted, though, that the supporting and fixing points of the Scott-Russel linkage must be aligned with respect to gravity. As a consequence, the main rod of the mechanism must be designed with gimbal-like shape, in order to leave the central volume free for the following stages. Another limit of the Scott-Russel linkage comes from the need to realize a slot joint for the bottom fixing points, which may introduce undesired friction phenomena.

Some potential advantages have been recognized in the implementation of a horizontal seismic isolation stage through the 3-D Roberts linkage [10–12], shown in Fig. 1, middle. With respect to the Scott-Russel linkage, the Roberts linkage has some notable advantages: the volume below the reference point is free and this allows for a simpler implementation; it only adopts revolute joints to be possibly implemented as flexure joints (which can be provided by thin wires), thus avoiding the effects of friction; finally, its compactness may be useful if the system must be enclosed in a vacuum chamber. A Roberts linkage-based seismic isolation stage has been operated with resonant frequency of 50 mHz and showed interesting noise rejection performance [2].

Starting from these considerations, the present work describes a novel optimal design and development method of a two DoF suspension system for inertial isolation based on a Roberts linkage mechanism. The novelty of the design approach hereafter described comes from the optimality criteria adopted for maximizing the lower boundary of the isolated bandwidth (i.e. reducing the resonant frequency) and at the same time ensuring the system isotropy, which means equal noise rejection regardless the direction along the plane orthogonal to the gravity. This is made possible by optimizing the *shape* of the virtual surface described by the mass suspension point (see Fig. 1, right), which constitutes the potential dwell, in which the isolated system is constrained to move by the linkage. A relevant novelty is also present in the verification approach, through which the actual surface generated by the suspension point is assessed and compared with the required one. In particular, assessment information are not limited to the resonant frequency, but also involve the overall kinematic/dynamic performance of the linkage, its isotropy, and possible machining and assembly errors.

2. Conceptual design

The 3-D Roberts linkage is a spatial mechanism constituted by a rigid body constrained to the ground through three rods that have revolute joints at both ends, as shown in Fig. 2. In order to adopt such linkage as a 2-DoF inertial isolation system, its stiffness has to be large in vertical direction z and minimal along the lateral x and y directions, since low stiffness means low resonant frequency.

It may be demonstrated [13] that such dynamics requirement on stiffness corresponds to a kinematic constraint on the curvature radii of the trajectories of the linkage center of mass, and of the test-mass suspension point (called R_g and R_v, respectively; see Fig. 1, right). Furthermore, when the linkage mass is negligible respect to the test-mass, the dynamics becomes dominated by R_v.

In other words, if a nearly planar, curved upwards surface is generated by the attachment point of the test-mass suspending wire, P_4, the system is dynamically quasi-equivalent to a composed pendulum, where the virtual wire (R_v) may be much longer than the second real wire. In particular, the flatter is that surface, the longer results the virtual wire R_v. The surface generated by the possible positions of the reference point of the suspended mass will be hereafter referred to as the *virtual surface*.

The target linkage dynamic behavior (its resonant frequency) is thus obtained by optimizing its kinematic performance (the flatness of the virtual surface). As far as the design of the prototype is concerned, a target resonance frequency equal to 300 mHz has