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Dynamics of dispersive long waves in fluids

Chao-Qing Dai [a], Yue-Yue Wang [a], Anjan Biswas [b,c,*]

[a] School of Sciences, Zhejihang Agriculture and Forestry University, Lin'an, Zhejiang 311300, PR China
[b] Department of Mathematical Sciences, Delaware State University, Dover, DE 19901-2277, USA
[c] Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah-21589, Saudi Arabia

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ABSTRACT

We obtain many different variable separation solutions for (2+1)-dimensional variable coefficient dispersive long wave equation by means of five different methods, including the multilinear variable separation approach, the projective Ricatti equation method, the extended projective Ricatti equation method, the extended tanh-function method and the improved tanh-function method. However, by careful analysis, we find that variable separation solution obtained by the multilinear variable separation approach includes all variable separation solutions obtained by other four direct methods. Thus variable separation solution for (2+1)-dimensional variable coefficient dispersive long wave equation exists a uniform form. Based on this uniform variable separation solution, we discuss the completely elastic interaction between foldons, the non-completely elastic interaction between bell-like semi-foldon, peaked semi-foldon and foldon, and the completely non-elastic interaction between bell-like semi-foldon and peaked semi-foldon. These results are helpful to analyze more precisely nonlinear and dispersive long gravity waves traveling in two horizontal directions, such as the bubbles on (or under) a fluid surface and folded waves in various ocean waves.

1. Introduction

As self-localized robust and long-lived solitary waves that do not disperse and preserve their identity as they travel through a medium, nonlinear wave excitations and solitons are ubiquitous in Nature and play increasingly important role in the investigation of many natural science from chemistry, biology, mathematics and communication to almost all branches of physics such as fluid dynamics, plasma physics, field theory, nonlinear optics and condensed matter physics. Recently, the wave-particle duality and intrinsic ("hidden") degrees of freedom have been shown up in the soliton scattering on potential barriers and wells (Belyaeva and Serkin, 2012).

In modern soliton theory, the variable separation approach (VSA) is a crucial and powerful mean to obtain abundant and general solutions of NLPDEs. To our excitement, many kinds of "variable separation" procedures have been presented recently. For example, the multilinear variable separation approach (MLVSA) was established firstly in 1996 for the Davey–Stewartsen equation (Lou and Lu, 1996). Wen (2011) and Shen and Jin (2011) obtained variable separation solutions of some NLPDEs via the Bilinear method. Moreover, many direct methods, which used to obtain traveling wave solutions of NLPDEs, have been successfully extended to derive variable separation solutions. For example, since Zheng et al. (2004) used the extended tanh-function method (ETM) to realize variable separation for the (2+1)-dimensional Broer–Kaup–Kupershmidt system, many authors generalized this method to obtain variable separation solutions for other (1+1)-dimensional (Dai et al., 2006a; Zhu et al., 2006), (2+1)-dimensional (Dai et al., 2008; Fang et al., 2005a; Ji and Lü, 2005; Xu and He, 2006) and (3+1)-dimensional (Dai et al., 2006a; Zhu and Zheng, 2007; Zhu et al., 2008) systems. Then, an improved ETM was presented and applied to obtain variable separation solutions for NLPDEs (Dai and Wang, 2009; Ma and Fang, 2009; Ma and Zhang, 2010). Moreover, the projective Ricatti equation method (PREM) (Dai and Ni, 2006b; Zhu and Ma, 2008) and extended PREM (El-Sabbagh et al., 2009) were constructed to realize the variable separation to NLPDEs, respectively.

So far, one can obtain seemingly different variable separation solutions for NLPDEs via MLVSA, ETM, improved ETM, PREM and extended PREM. However, by careful analysis, we find that these different variable separation solutions obtained via MLVSA, ETM, improved ETM, PREM and extended PREM are uniform. To illustrate this point, we apply these five different methods to the following (2+1)-dimensional variable coefficient dispersive long

* Corresponding author at: Department of Mathematical Sciences, Delaware State University, Dover, DE 19901-2277, USA. Tel.: +1 302 857 7913.
E-mail address: biswas.anjan@gmail.com (A. Biswas).

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wave equation (vcDLWE) (Zhang et al., 2002)

$$u_{ty}+\alpha(t)h_{xx}+\frac{\beta(t)}{2}(u^2)_{xy}=0, h_t+\lambda^2\alpha(t)u_{xxy}+\beta(t)(hu)_x=0, \quad (1)$$

where $\alpha(t)$ and $\beta(t)$ are two functions of variable $\{t\}$, and λ is an arbitrary nonzero constant. When $\alpha(t)=\beta(t)=1$, this equation is the celebrated (2+1)-dimensional DLWE, which was used to model nonlinear and dispersive long gravity waves traveling in two horizontal directions on shallow waters of uniform depth. The (1+1)-dimensional DLWE ($y=x$ in (1)) is called the classical Boussinesq equation (Musette and Conte, 1994). The (2+1)-dimensional DLWE has been extensively studied by many authors (Chen et al., 2012; Lou, 1994, 1995; Ma and Zheng, 2006; Paquin and Winternitz, 1990; Tang et al., 2002; Wen, 2012). Paquin and Winternitz (1990) presented that the symmetry algebra of DLWE is infinite-dimensional and possesses the Kac–Moody–Virasoro structure. The more general W_∞ symmetry algebra of DLWE has also been given in Lou (1994). In Lou (1995), both the direct method and the non-classical Lie approach were applied to reduce the (2+1)-dimensional DLWE. Moreover, abundant localized coherent structures were discussed and investigated. Dromion, ring soliton and peakon were derived by Tang et al. (2002). Two classes of fractal structures were obtained by introducing appropriate lower-dimensional localized patterns and Jacobian elliptic functions (Ma and Zheng, 2006). Non-completely elastic interactions between semi-foldons were discussed (Chen et al., 2012). Fission and fusion interaction phenomena were studied analytically (Wen, 2012).

As we all know, one of the outstanding characteristics for solitons is their completely elastic interactions between solitons, namely, their amplitudes, velocities and shapes do not undergo any change after the nonlinear interaction. However, recent investigations show that for some special solutions of certain (2+1)-dimensional systems, the interactions between solitonic excitations were not completely elastic since their shapes or amplitudes were changed after their collisions (Chen et al., 2012; Dai and Wang, 2009; Tang et al., 2002). Even the completely non-elastic interaction (fusion phenomena) has been reported for (1+1)-dimensional (Ying, 2001) and (2+1)-dimensional (Dai and Chen, 2006c; Wen, 2012) systems, that is, some solitons fuse one soliton after the nonlinear interaction. Actually, the solitary wave fusion phenomena have been observed in many physical fields like plasma physics, nuclear physics and hydrodynamics (Serkin et al., 2001).

Single-valued line solitons used to analyze nonlinear and dispersive long gravity waves traveling in two horizontal directions. For example, we can use them to describe roughly the tidal bore on the Qiantang River in East China's Zhejiang province. However, the tidal bore is too complicated to use only single-valued functions to analyze the dynamical behaviors of water waves. More precisely, we can use multi-valued functions to describe them. In 1+1 dimension, much effort (Li and Zhang, 2009) has been focused on multi-valued localized excitations such as loop-soliton solutions since Konno et al. (1981) first reported them in a nonlinear oscillation model of an elastic beam with tension. In 2+1 dimension, since Tang and Lou (2003) first extended the (1+1)-dimensional loop-soliton solutions into variable separation solutions in 2+1 dimension, multi-valued (semi-) foldons have attracted a great deal of interest (Dai and Ni, 2006b; Lei et al., 2013) because very complicated folded phenomena such as folded protein (Trewick et al., 2002), folded brain and skin surfaces, and many other kinds of folded biologic systems exist in the real natural world (Goodman et al., 2002). The simplest multi-valued (folded) waves may be the bubbles on (or under) a fluid surface. Various ocean waves are really folded waves, too. Of course, at present stage, it is impossible to make satisfactory analytic descriptions for such complicated folded natural phenomena. However, it is still worthwhile to start with some simpler cases. In this paper, based on variable separation solution, we discuss completely elastic interaction, non-completely elastic interaction and completely non-elastic interaction between special multi-valued dromion, peakon and foldon.

2. Variable separation solution via MLVSA

By means of the standard truncated Painlevé expansion (Tang et al., 2002), one has a special Painlevé–Bäcklund transformation for differentiable functions u and h in (1)

$$u=2\lambda\mu(\ln f)_x+u_0, \quad h=2\lambda\mu(\ln f)_{xy}+h_0, \quad (2)$$

where $f=f(x,y,t)$ is an arbitrary differentiable function of variables $\{x,y,t\}$ to be determined, and u_0, h_0 are arbitrary seed solutions satisfying the vcDLWE (1). In usual cases, by choosing some special trivial solutions, we can directly obtain the seed solutions. In the present case, it is evident that Eq. (1) possesses trivial seed solutions $h_0=0, u_0=u_0(x,t)$ with an arbitrary function $u_0(x,t)$.

Inserting (2) with the seed solutions into (1) yields an identical trilinear equation

$$[f^2\partial_{xy}-f(f_x\partial_y+f_y\partial_x+f_{xy})+2f_xf_y][\mu\beta(t)f_{xx}+f_t+\beta(t)f_xu_0]=0, \quad (3)$$

with $\alpha(t)=\mu\beta(t), \lambda=1$ and an arbitrary constant μ. If Eq. (3) has the linear superposition solution

$$f=Q_0(y)+\sum_{i=1}^{N}P_i(x,t)Q_i(y,t), \quad (4)$$

with arbitrary functions of indicated arguments, we have the following simple variable separated equations

$$P_{kt}+\mu\beta(t)P_{kxx}+\beta P_xu_0+\sum_{l=1}^{M}C_{kl}(t)P_k=0, \quad (5)$$

$$Q_{kt}-\sum_{l=1}^{M}C_{kl}(t)Q_k=0 \quad (l=1,2,\ldots,M), \quad (6)$$

where $C_{kl}(t)(k=1,2,\ldots,N, l=1,2,\ldots,M)$ are arbitrary functions of $\{t\}$.

Substituting all the results into (3), we obtain a general variable separation solution for the vcDLWE

$$u=2\mu\frac{\sum_{k=1}^{N}P_{kx}Q_k}{Q_0+\sum_{k=1}^{N}P_kQ_k}+u_0, \quad (7)$$

$$h=2\mu\left[\frac{\sum_{k=1}^{N}P_{kx}Q_{ky}}{Q_0+\sum_{k=1}^{N}P_kQ_k}-\frac{\sum_{k=1}^{N}P_{kx}Q_k(Q_{0y}+\sum_{k=1}^{N}P_kQ_{ky})}{(Q_0+\sum_{k=1}^{N}P_kQ_k)^2}\right], \quad (8)$$

where P_k and Q_k satisfy (5) and (6).

If selecting $N=M=1, Q_0=q(y), \{P_1,Q_1\}=\{p(x,t),1\}$, then (4)–(6) have the following form:

$$f=p(x,t)+q(y), \quad (9)$$

$$p_t+\mu\beta(t)p_{xx}+\beta(t)p_xu_0=0, \quad C_{11}=0. \quad (10)$$

Since u_0 is an arbitrary seed solution, from Eq. (10) we can view p as an arbitrary function of $\{x,t\}$, then the seed solution u_0 is fixed as

$$u_0=-\frac{p_t+\mu\beta(t)p_{xx}}{\beta(t)p_x}.$$

Therefore, the special variable separation solution has the following form:

$$u=\frac{2\mu p_x}{p+q}-\frac{p_t+\mu\beta(t)p_{xx}}{\beta(t)p_x}, \quad (11)$$