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Freeform surface filtering using the lifting wavelet transform

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ABSTRACT

Texture measurement for simple geometric surfaces is well established. Many surface filtration techniques using Fourier, Gaussian, wavelets, etc., have been proposed over the past decades. These filtration techniques cannot be applied to today's complex freeform surfaces, which have non-Euclidean geometries in nature, without distortion of the results. Introducing the lifting scheme opens the opportunity to extend the wavelet analysis to include irregular complex surface geometries. In this paper, a method of filtering those complex freeform surfaces presented by triangular meshes based on the lifting wavelet has been proposed. The proposed algorithm generalises the traditional lifting scheme to any freeform surface represented by any type of triangular mesh; regular, semi-regular or irregular mesh. This technique consists of five major stages; split, predict, update, simplify (down-sampling) and merge (up-sampling). All of these techniques are discussed and explained in the paper. Results and discussion of the application of this method to simulated and measured data are presented.

1. Introduction

Over many years, the theory of measuring and characterising ordinary simple surfaces such as planes, spheres and cylinders has been developed [1–5]. Indeed, many research papers and industrial standards have been published to describe the measurement and characterisation of such surfaces [3–10]. However, with the development of science and technology, more and more complex surfaces are being produced which, unlike the conventional surfaces, have no axes of rotation and no translational symmetry and could have any shape or design; such complex surfaces are called freeform surfaces.

In the case of simple geometries, surfaces are regarded as a continuous function that defines a height value over a planar domain. This planar domain is Euclidean in nature; has zero curvature. In this case many well-established methods for studying the surface data are available for example Fourier analysis, Gaussian filters, etc. However, in the case of freeform surfaces the underlying domain is no longer a plane, instead it will have positive or negative surface curvatures according to the surface's geometry and thus the underlying domain is non-Euclidean in nature.

Characterisation and parameterisation of surface texture on such freeform geometries is very challenging and requires re-thinking each characterisation step of the simple surfaces. Traditionally, the characterisation and parameterisation of surface texture is carried out using the four major steps namely: surface sampling and representation, decomposition and filtration, texture representation and mapping and finally characterisation and parameterisation as shown in Fig. 1.

Moving from simple geometries to complex freeform geometries, many of the traditional techniques used to perform any of the tasks shown in Fig. 1 start to fail. Therefore, new theories and tools that can cope with the new emerging surfaces are required.

Surface decomposition and filtration is an essential step of the texture characterisation system. During the last decade, decomposition and filtration techniques for simple surfaces have been comprehensively investigated and many algorithms based on Fourier, Gaussian, Spline and wavelet techniques were proposed and became the industrial filtration standards [6–10]. Unfortunately, all of these techniques are designed to decompose and filter Euclidean surfaces, so most of these techniques fail to filter freeform non-Euclidean surfaces.

Very recently, our research group have proposed a new filtration technique for freeform surfaces, which are represented by triangular meshes, based on solving the diffusion equation formulated by using the Laplace–Beltrami operator on that surface [11].

In this paper, we provide an initial investigation of applying the lifting wavelet for freeform surface filtration. The power of this filtration technique is that it is capable of filtering any type of freeform surfaces represented by a triangular mesh. We investigate different methods of building the lifting scheme and the results discussed and presented.

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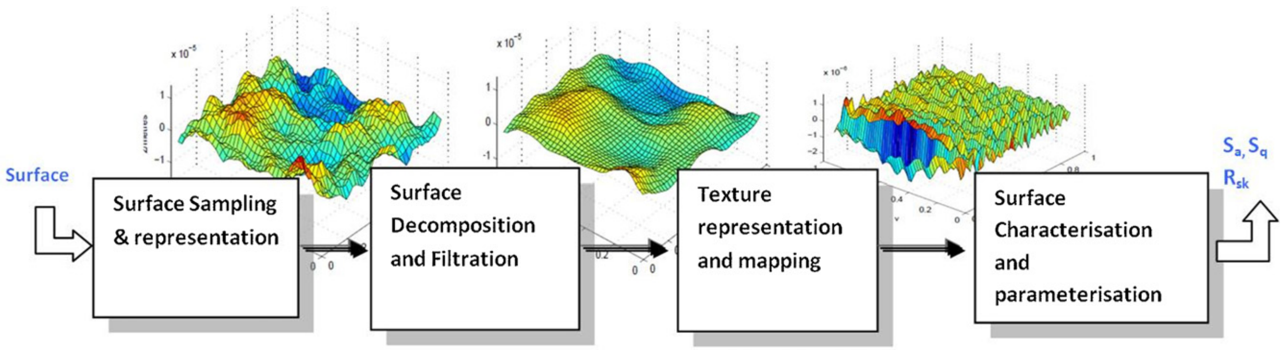

Fig. 1. Texture characterisation and parameterisation.

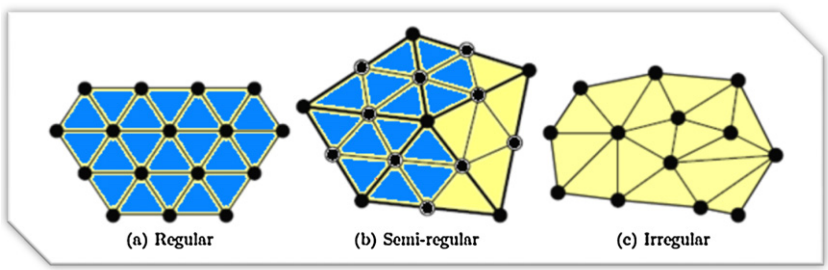

Fig. 2. Different types of triangular meshes: (a) regular; (b) semi-regular and (c) irregular.

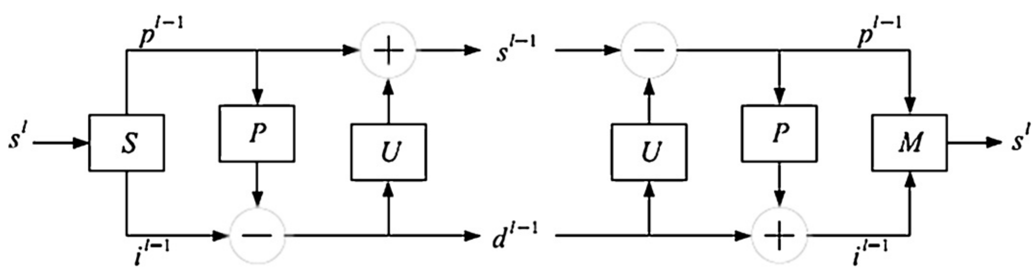

Fig. 3. The lifting scheme decomposition and reconstruction process.

This paper is organised as follows: Section 2 discusses different techniques to represent a freeform surface; Section 3 gives a brief introduction to the lifting scheme and second generation wavelets; a brief review of wavelets and multi-resolution analysis on such surfaces is shown in Section 4; Section 5 details the proposed lifting algorithm on freeform surfaces and the results of the algorithm is shown in Section 6; and finally, the conclusions and future work is discussed in Section 7.

2. Representing freeform surfaces

Traditionally, surfaces are represented as height values over a plane. This type of representation is only valid for simple Euclidean surfaces. This representation method enables researchers to successfully apply different processing techniques, such as Fourier analysis, wavelet decomposition and Gaussian filters, to analyse surface data. Freeform surfaces cannot be represented using this

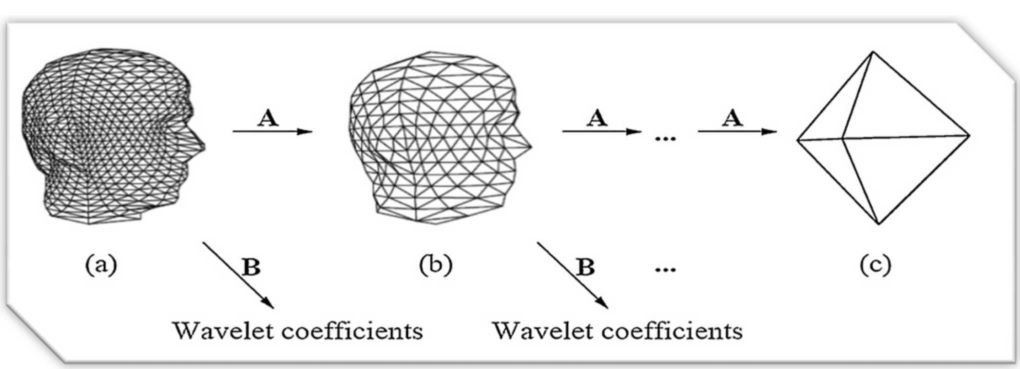

Fig. 4. Decomposition of 3D mesh into approximation and details as proposed by Lounsbery [19].